NEW ECONOMY Section

SOCIAL CONVENTIONS,DUALITY, CIRCULARITY, SET THEORY ABOUT FOUNDATION OF LYONG AND VIOLENCE

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Abstract. *Both the behavior of societies and post-modern geopolitics are subject to an engineering of chaos. The current chaos is also mainly due to the closure of the planet upon itself and the hyperbolicity which results from the associated feedback within inside relationships. This chaos is enhanced by the demography in poor countries and the exponential evolution of the social networks. The mastering of this chaos requires having the tools of thought allowing us to face ambient nihilism by rediscovering a rationality adapted to its internal complexity. We show here how mathematics, which is essentially confronted with the same class of problems as anthropology, can approach this question [GOF37,45], [CHG10] [LOG11], [GRG68], [SEM13] We will back the arguing on the notions of convention, of monotony of hierarchy and fix point in social environment.*

Keywords: *Convention. Circularity, Foundations , Fix Point.*

1. Introduction

Both the behavior of societies and post-modern geopolitics have been subject, at least since 2007, to an engineering of chaos which now affects including the G20 economies [DAG19]. Besides the hyperbolic effects due to the closure of all the social networks upon themselves we can read the evolution towards chaos as the result of engineering carried out, among others reasons, by certain countries (small or big), for getting geopolitical influence. It is by universalizing reasoning through the understanding of the cosmos, therefore by giving itself infinite limits, that the Italian renaissance and Arab scholars succeeded in giving between the 15th and the 17th century a status to truth and therefore in constructing a new consciousness of the human being in a flat world [KRI75]. This one found all its fullness in the age of enlightenment then gave birth to revolutions and a conquering capitalism through its innovations and concepts. A reverse movement is now underway and obscurantism is back. Some great countries, dreaming about their "glorious" history, but unable to realize this dream, search of scapegoats for failing (economic, social or geopolitical) an push to war, namely the ultima limit of chaos.

However, this situation is perhaps only the symptom of a much larger problem. The chaos of the world can in fact be understood as a consequence of the closure of the planet on itself and of the hyperbolicity which results from the multiple loops associated with the international interdependence which this closure requires to achieve. Far from the standard rationality which underpins the curriculum of elites formatted by differential calculus, or even simple arithmetic of accounting, mainly depoliticized, a chaotic environment requires having the necessarily complex tools of thought to deal with (i) the cognitive nodes imposed by the closures mentioned, (ii) the anxieties of the people induced by the insecurity (incompetence) of the elites in the face of reality (iii) the nihilism of *a lying slave that only the whip can move, (no goodness* [THO56], nihilism maintained by despotic powers playing with the vanity of people and the fantasy of “glorious” history, most of the time bloody.

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Faced with the intricacy of causes and effects, the rationalized use of probabilities, statistics and the reference to the sciences of pure chance are essentially unconvincing because they are mainly ineffective in action. By using the concept of chance, the elites then lose all authority. Then they naturally take refuge in clericalism or even authoritarianism [WEB59]. Faced with the death drive that chaos gives rise to, the life drive should be able to be rebuilt by marginal elites capable of designing renewed rationality, adapted to the complexity of a world now as limited as that of an ocean liner without commissioned harbor, living only with finite resources while challenged to Shakespearean storms. Any authority disappears.

In order to illustrate the project of rebuilding a cognitive authority by starting within the chaos, the following text recalls the way in which, faced with the crisis of foundations, mathematics, has built step by step a capacity to go beyond the theory of sets, capacity which today, establishes our ability for facing the paradoxes which justify, for some, the will of going to the overall chaos and its anthropological limit expression : the war[1]. This text only evokes a first stage of the project that after Boltzmann and Prigogine [STE97] among others we are pursuing to face more generally the deadly irreversibility of time. To illustrate the point, we will rely on the notion of convention, duality, chain, monotony and hierarchy.

2. A dual Reality: the notion of convention

Unlike standard physical systems, complex systems, tying together a multiplicity of tangled causes and effects according to Gordian knots, do not respond to linear laws as commonly taught at university where subjects and objects are distinct. The understanding of these systems[2] necessarily responds to specific rules, developed step by step in a circular and most often recursive manner that few researchers consider, because they require a very high critical sense that is all the rarer as it operates in bureaucratized institutions. How can we find a universality of our representations for such systems? That is the question. Among others [BOU00], [BAI09] ,[SML13], [PRI79], [ROV12], [DEG15] the authors' work shows that the reality with which the blind spots of normal science [KUH85] confront us is made up of a field of correlated singularities which should be mastered. Their complex relationships require the implementation of adjunctions and dualities capable of deploying (dynamically) these singularities on varieties whose contortions can exceed our capacities of representation if not our understanding. As shown for example after Prigogine [STE97], Yuri Babenko [TABA96] in revolutionary and highly original work, our skeptical attitude towards the problems generated by the complexity and hyperbolicity of our current environment, must almost certainly be associated with the properties of divergent series - more generally doubly divergent - whose characteristics can only surprise the physicist accustomed to equilibrium thermodynamics, Hamiltonian mechanics, and the L^2 geometry of Hilbert spaces [STE97]. The time parameter plays a central role in our ability to bring out and deploy singularities which interest us..

[1] Highlighting the paradox requires clearly separating truth and lies. Information warfare, lethal by induced effect, is based on mixing true and false information in the same expression. An archetypal example of this belligerent mixture can be found for example inhttpS://www.bfmtv.com/replay-emissions/22h-max/alexander-makogonov-porte-parole-de-l-ambassade-de-russie-en-france-poutine-est-beaucoup-plus-fort-ce-soir-18-03_VN-202403180958.html

[2] Exponential growth of information technologies; growth of information exchanges assisted by neural networks and questioning of rationality and the truths which justify them; international division of manufacturing labor; greed of economic actors and the death drive of capitalism; erratic structures of transactional and financial fluctuations; demographic growth (2 billion more human beings in 50 years); lean management etc.

Beyond deterministic fantasies and statistical eases, rational thought in chaos must therefore be constructed by walking. It is defined step by step according to the circumstances, but backed by complex chains and sequences, ordered according to values, principles, axioms, each of them being indexable, index which sole can give substance to a new kind universality. Thid one should replaces states functions and equilibrium sets associated to a primitive approach. The path between the anterior and posterior (Socrates) left and right etc , namely any duality, must nowbe taken into account into path integrals.

Even with a map and a compass, tracing a path in the mountains requires paying attention to where you put your foot, knowing the amplitude of your step and his altitude (Local), but also of knowing where you are coming from. (Memory), where we want to go and how long will be the time(Global). The important thing is not only the path taken but the whole set of the possible and impossible paths which, open to oneself at each moment, and beyond any geodesic optimum, are infinite in number. The scientist of tomorrow should be a mountaineer a hicker and no longer a boatman. All geodesic or optimal approaches standardized by the passed physicists considering ultimately states of equilibrium (deterministic or statistical) are excluded here. Complex systems do not respond to any equilibrium properties. We must design fundamentally the concentration of measures within non-equilibrium representations that may be very far from the set theory even if these last formats our usual intuitions, then to much trivial.

On a historical level, complexity prevents us from having a Hegelian vision. There is no more direction, signification of the history than of authority of the master versus slaves [HEG07]; at least until a charming and efficient power is reached. In this context, -which authors express by means of fractional operators-, the present moment is a condensate of past and future, of right and left, of representation and will, [SCH88]. The present is dual, thick and opens not onto the uncertain but onto the irreversible [STE96]. The fertility of this present moment is more or less important depending on the more or less tragic nature of the context[3]. Both the absence of universality and the coming back to personal responsibilities generate existential anxieties in individuals, all the more difficult to accept by human societies as they are weakened by food, biological and/or cultural insecurities or even, height of horror, by falling in physical insecurity under pressure from militias and mafia gangs. By erasing the complexity, the tyrants and other Satraps, whether already established or budding, take advantage of these fragilities to impose their power on zombified peoples, in particular by constructing fantasized mythologies which are intended to be rewarding and reassuring. Against all evidence, this assurance is essentially due to a representation simplified by linearization and determinism (even statistical) , that is to say conforming to immutable balances or historical permanences, most often purely imaginary. In order to understand the mechanisms of voluntary slavery [LAB76] of peoples in need of hope and their capacity to constrain the brake which will stop the machine of alienation, we can read the work of Henri David Thoreau on Civil Disobedience [THO56], book released in 1894 from the anonymity into which he had fallen, by the great Russian writer Leo Tolstoy. At that time, *Tolstoy knew what it meant to break with standard conventions and submission to tyranny starting with that of the boyars.*

The following study is, at least partially, the axiomatized review of an approach to the notion of convention, both approached from a philosophical point of view, then from a mathematical point of view. Our aim, on a specific axiomatizable case, to point out the characteristics implied by the ruptures of the conventions such as we can observe currently for example in the

[3] Tragedy can be defined as the impossibility of prioritizing good and evil, irremediably entangled by the context.

geopolitical conflictual relations. If we consider that philosophy and literature are characterized by the pivotal role played by strange attractors, mathematics and the human sciences find themselves linked by the properties of an induced or maintained "chaos" which can be described as “deterministic”, insofar as it responds to a determined policy. This note is based on a communication made on behalf of the ESEIA engineering school at the Rochebrune Scientific Days (JSR’23) by one of the authors who is also an expert in mathematical science (P. Riot).

Among other works, the studies refer to a philosopher of the analytic tradition, named David K. Lewis [LEW69], a student of van Orman Quine. We can consider that Lewis is in some ways, the ultimate systematic metaphysician of the twentieth century, perhaps the most important. He was born in 1941, and died young in 2001. D. K. Lewis was the promoter of a school of thought, essentially present and affiliated with the analytical tradition called modal realism. He published the following major books: 1) Convention (1969); 2) Counterfactuals (1970); 3) On the plurality of worlds (1986); 4) Parts of classes (1991). It is the third work cited which established his definitive reputation. He affirms, among other things, regarding the plurality of worlds: *[…] all that exists in the world is a vast mosaic of local affairs of particular facts, just one little thing and then another and so on. We have geometry: a system of points with external relationships like space-time distance between them. […] At these points local qualities are associated : perfectly natural intrinsic properties that need nothing larger than a point at which to be instantiated. Object is a quality arrangement. And that's all. There is no difference without difference in the arrangement of qualities. Everything exists only according to this arrangement..* These lines show how difficult it is to identify an universality by starting from a modal realism.

The book, which is the source of the presentation at the scientific Roquebrunes meeting is the book about the conventions. This short work comes from a thesis up held by the author under the direction of the famous logician and philosopher van Orman Quine, whose legend says that he intellectually disagreed with the thesis defended by his student, who did not clearly did not lack the insane courage to oppose his famous thesis director, so breaking with standard conventions by illustrating in vivo the content of his work! As recent geopolitical developments show, it is difficult to regain such courage while the breaking of conventions reflects, more often, a mere anthropological step backwards [TOD23] by returning to previous conventions (against reforms, restoration, wokism, etc.).

One of the main aims of the Lewis first book is to support and reevaluate Carnap's arguments in favor of analyticity: all knowledge is based upon a language and a local living. It is a question of rehabilitating the analytical/synthetic distinction in the face of Quine's famous refusal concerning the distinguo between observation and representation, posture which explains the conflict referred above. In demonstrating how speech carries modal meanings, Lewis argues that conventions are games of coordination. As I. Stengers (STE96) shows with regard to Boltzmann's work, this coordination is imposed by an absence of “precise determination of object" when its relations with its outside are not defined. The application of game theory to the analysis of conventions, understood as a coordination problem, has been inspired by Thomas Schelling, professor of economics at Harvard who used game theory in his studies on conflict strategies. , work which earned him he Sveriges Riksbank Prize in Economic (2005).

3. About circularity : the role of cycles

David Lewis discovered a major source of circularity in human activities. He analyses it in his book Convention. In summary, all social institutions, from languages, to laws, or to the rules of

use applied in common life (example of the highway code) are based on conventions. They are shared by a community, which establish the rules of playing according to *cyclic agreement*. We can then expect that any breakdown in circularity will become a source of social violence (if, for example, I decide in France to apply the English highway code). Observation of our surrounding allow us to note at each moment the presence of cycles, for example, the flow of weeks or seasons or meal. Such cycles abound not only in the physical world, but also in the economic worlds (Kitchin (3-4 years); Julgar (8-10 years), Kuznets (15-25 years) Kondratiev (20-30 years)), in biology (let's mention the heartbeat or the functioning of the digestive system) and psychological (bipolar behaviour), etc. Social interactions are also systematically marked by reciprocal activities generating circularities; an interesting example is provided by the relation master and slaves [LAB76], [HEG07], boyar and mujiks (Tolstoy, Troyat), for which the circularity is ontological. All these cycles and their correlations are associated with the practical efficiency of the systems at stake. Measuring this efficiency involves the need to put in place tools for measuring change. They are based on a reference called the unit of time. Its determination is mechanical : clepsydra, pendulum , clock, etc. which are based upon the use of the irreversibility [STE96]. The theoretical question posed by the arrow of time and therefore the irreversibility of time [WOF23], [ROV15,17] which passes in open systems, is to know under what conditions this unit of time can be the same as that of physical phenomena whose irreversibility is observed and therefore, what are the conditions for the conjunctions of different cycles. To give rise to some objectivity they must respond to entire phases and therefore to rules of perfect divisibility. Here we see fundamental mathematical questions emerging within the human sciences.

What does it mean for a company to share an agreement or for employees to accept hierarchical authority? people who accept an agreement, denoted C in the following, behave in an obligatory manner which makes it possible to objectify the said agreement. Lewis highlighted the fact that what allows C to act as a convention comes from the fact that those who accept C also accept that C is accepted as a convention by others. The status of a convention is then legitimate if it is circular. Let's consider an easy-to-understand example. Let us denote by C the convention according to which pedestrians or motorists travel on one side of the street, say on the right. Accepting C for each of us involved in a flow of traffic means that we know that we must move to the right; but it also means that we expect others to do the same. In simple terms, we accept that C is a shared convention. This formulation is circular, since part of our understanding of C consists of others also involved in the same circulation process understanding the same thing. The analysis of the behavior of drivers with regard to respecting the highway code is largely based on the confidence that fellow citizens also respect the same rules. If one of us violates the rule, or the law, we also violate the expectations of other drivers around us. At the geopolitical level, the same is true for internationally recognized borders. If one of the countries considers that the notion of border no longer applies to it, that its country is without borders, or that border is defined by its tanks, the international community finds itself upset, the circularity of convention disappears and the circularity of violence replaces the circularity of cooperation.

The notion of circularity is very common in philosophy. It coincides with numerous logical and semantic paradoxes. Without pretending to conduct an in-depth philosophical analysis, let us quickly examine an example. Descartes' founding statement, "Cogito, ergo sum"; for him it is a question of questioning everything that can be questioned until reaching an element of certainty from which everything can be built. he looks for a basis for an universality. In this search, the only thing that cannot be called into question is then the fact of thinking, that is to say, in this case, of carrying out the questioning that I am currently carrying out. Once this is acquired, Descartes believes he is able to construct a coherent philosophy. The statement

“Cogito, ergo sum” is intended to be an irrefutable argument based on an undeniable premise. This sentence can be reformulated as follows: I am thinking this thought, I cannot doubt it since doubting requires that I think. The reasoning is clearly circular. The same goes for the notion of authority. For example that of the lord on the serfs; of the tribune over the crowd; of the wise man on his disciples, the influencer or guru on his followers.

The same goes for the status of language. When we speak French in front of an audience, we make the tacit assumption that each of the people present (at least most of them in a concrete situation) knows this same language. The philosopher and logician Grice suggests analyzing the situation as follows: (a denotes a speaker, S denotes a spoken sentence) “a means something through S” is analyzed as follows: a intends by stating S to produce a certain effect in the audience by recognition of this particular intention. In other words, the speaker ensures that his audience becomes aware of, and reacts accordingly to, his intention through what he says; also the intention I of a is circular, since the intention I to do something fundamentally depends on I itself. The process is teleonomic.

Analysis of the emergence of common knowledge, which constitutes the heart of the reasoning established by Lewis in the work considered above, was extended by the study of Clark and Marshall [CLA81]. In Lewis's vision, shared knowledge is necessary for an agreement to be established between the members of the group: common knowledge between b, c and t of the same fact σ emerges from a situation or event s that everyone observes and this clearly shows that σ is an established fact and that everyone, b, c and t, knows or recognizes the facts of s. Let us introduce the symbolization: $s \vDash \tau$ indicates that τ is one of the facts attested by the situation s.

$$s \vDash \tau$$
$$s \vDash (b \text{ know that } s)$$
$$s \vDash (c \text{ know that } s)$$
$$s \vDash (t \text{ know that } s)$$

The underlying idea is that shared knowledge comes from the perception of an event which attests to the fact in question, but at the same time attests to the awareness by the participants. other agents of the situation in question. It is clear that this formalism is circular. We see here the huge risk represented by lying or the use of confusion in social relations. The circularity required for partnership is disrupted. If we add that anaphora in linguistics is also a source of self-referentiality, we foresee the danger if authority is preempted by an evil being (Calligula, Néron, Scarpia, Hitler, Staline …[VHO19]); herein step by step the circularity of the violence replaces the circularity of partnership as new convention just before conventional fear. In theoretical economics, game theory is heavily used; in fact in most cases, there is an implicit assumption that each player knows the information structure of the other players. If we want to formally explain this fact, a circularity emerges.

4. Emergency based on paradoxes

Circularity, which has just been identified, also plays an important role in information processing systems, in formal computing and even in cognitive science. The notion of the state of a system such that the system in question returns to a given state on a regular basis. This notion of state is a fundamental aspect of the notion of the Turing machine. Circularity played a fundamental role in the emergence of modern logic from the major works of Gottleb Frege,

and in the foundational approach to mathematics from the equally major works of Georg Cantor. We propose to report certain well-established and known elements of a logical and mathematical nature, thus making the link with Lewis' analysis of which we have restored some items previously.

It is at this step that paradoxes like that of the liar of Epimenides appear (all Cretans are liars. If a Cretan affirms that he is lying it is impossible to know if he is lying or if he is telling the truth) or that of Russell (the hairdresser who does not shave himself. If he shaves he does not shave. If he does not shave he must shave). Undecidability is one of the consequences of certain forms of internal circularity.

Russell, who before Gödel has broken the Frege's project of flawlessly ensuring mathematics through number theory, promoted a formal approach aiming to exclude any vicious circle. Tarski took up this objective by requiring us to speak of the truth of statements in a language but by calling on a metalanguage. This new language can thus generate its own metalanguage, generating an infinite hierarchy of truth predicates. This approach was widely used, until the publication of an article by Kripke [KRI75] "Outline of a theory of truth". Below a canonical example from this article:

1) Most of Nixon's statements about Watergate are false
2) Everything Jones says about Watergate is true

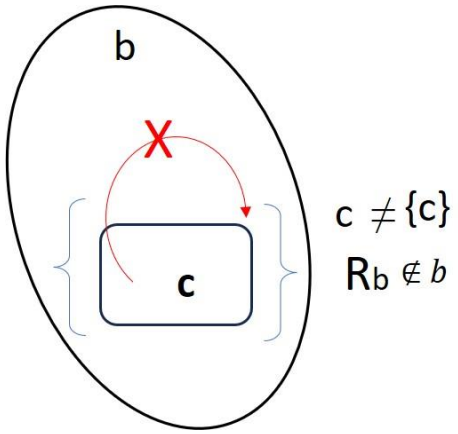
In many cases, the association of the two statements is non-paradoxical. However, there are certain circumstances or modalities which make this association paradoxical, for example: Jones states (1) only about Watergate, while Nixon says (2) then eight other statements relating to Watergate, half true and half false. What should we say about (2)? This statement is then true if and only if it is false.

As we recalled above, the problem has been known since antiquity with the Cretan paradox asserting that all Cretans are liars. The statement qualifies itself as a lie. If it's true, then he's lying. But if he is lying, then the Cretans are not liars, which means they are telling the truth. The logical arguing creates an puzzling contradiction. We owe to Aristotle a solution distinguishing between absolute truth and relative truth. He suggested that the statement could be understood as follows: "I speak the truth when I say that I lie." Truth then becomes relative to specific content. Circularity is here a key element of the problem which will only be understood in the 20th century with modal analysis. While our societies are based on the gold of data, we can observe the use of circular reasoning in information warfare globalized for instance through social networks . In fact, in circular reasoning, it is enough to mix true and false information for confusion to set in and for fading of the universal status of the truth to make way for simplistic alternatives "realities", like new attractors of sophisms.

Mathematically there are several formulations of the paradox now called Russell's paradox. The relevant formulation that concerns standard set theory concerns *non-reflective sets, in other words sets that are not members of themselves. According to Frege, every predicate admits a set for extension*. There is thus a set R grouping all non-reflexive sets: $R = \{x: x\ est\ non\ reflexif\}$. Then $R \in R$ if and only if R is non-reflexive, which means $R \notin R$. The hypothesis of exclusivity, that is to say according to the principle of the excluded middle, $R \in R$ or $R \notin R$, leads to a contradiction [4]. In this reasoning, we do not appeal to any other hypothesis relating

[4] Identity concept, homotopy and type theory.

to set theory than the fact that the predicate analyzed determines a unique set. In another way, we can consider the following theorem of first order logic which is independent of the content of the predicate $E : \neg \exists x \forall y [yEx \leftrightarrow \neg (yEy)]$. Russell summarized the context by stating the rule: "whatever involves all of a collection must not be one of the collection". We can easily escape from Russell's paradox: let any set b be understood as a universe, we can then form $R_b = \{c \in b: c\ est\ non\ reflexif\}$, R_b there is nothing paradoxical; the preceding reasoning leads to the only requirement : $R_b \notin b$.



Paradoxes are certainly a source of frustration, but they also prove particularly fertile for obtaining many important results. Let's take the following concepts for example:

- *Self-sufficiency*: A set of formulas $\mathcal{F}$ is said to be self-sufficient if $\mathcal{F}$ contains all the atomic formulas, their negation ($\neg$), and is closed, up to logical equivalence, under $\wedge$,$\vee$, (AND intersection and OR union) the related quantifications $\exists x \in y$ and $\forall x \in y$ and the free existential quantification $\exists x$ and the equivalence $Sat_n(\lceil\varphi\rceil, x_1,\ldots, x_n) \leftrightarrow \varphi(x_1,\ldots, x_n)$. The "Sat" operator here is the Satisfaction operator.

- *Recursion theorem*: for $\mathcal{F}$ a self-sufficient set of formulas, for everything $\varphi(x_0, x_1,\ldots, x_n) \in \mathcal{F}$ there exists a formula $\psi(x_1,\ldots, x_n) \in \mathcal{F}$ verifying: $\psi(x_1,\ldots, x_n) \leftrightarrow \varphi(\lceil\psi\rceil, x_1,\ldots, x_n)$. In other words, everything happens as in a vector space whose axes support the parameters x_k, k={1,2,..n}the formula is part of the set of predicates. There is recursion.

We then obtain a negative result directly taken from the liar paradox: For $\mathcal{F}$ a self-sufficient set of formulas, let Sat_0 be the satisfaction formula for the statements in $\mathcal{F}$, then $\neg Sat_0$ is not equivalent to any formula in $\mathcal{F}$. Indeed, by the recursion theorem, we obtain a statement φ equivalent to $\neg Sat_0(\lceil\varphi\rceil)$ assuming $\neg Sat_0$ in $\mathcal{F}$. This means precisely: $\neg Sat_0(\lceil\varphi\rceil) \leftrightarrow \varphi$, hence: $Sat_0(\lceil\varphi\rceil) \leftrightarrow \neg Sat_0(\lceil\varphi\rceil)$, which leads to a contradiction by the excluded middle principle. Consequence: no self-sufficient set of formulas is closed for negation. In particular, neither the set of first order formulas nor the second order formulas are self-sufficient.

Let us start from a self-sufficient set $\mathcal{F}$, for each index k there exists a formula $\neg Sat_0$ which is not a formula of $\mathcal{F}$: this agrees with Russell's paradox the set R_b is well formed, and it cannot belong to universe b. If $\mathcal{F}$ is the set of formulas Σ_n then $\neg Sat_0$ belongs to Σ_{k+1}, in other words the formulas Σ_{k+1} contain more information than those of Σ_k. This observation leads to an *arithmetic hierarchy*. Likewise, there is no simple first order formula $\varphi(v)$ such that for all

statements ψ, we have $\varphi([\psi]) \leftrightarrow \psi$ (theorem of the indefinability of truth established by Tarski). These results, which will be exploited among others by Kurt Gödel in his incompleteness theorem, put an end to the projects of Gottlob Frege and David Hilbert to flawlessly base mathematics on arithmetic at the beginning of the 20th century.

As we noted previously, information processing theory examines systematically the notion of circularity. Its formal treatment addresses the role of cycles within the axiomatic framework of the set theory which usually serves not only as the basis of classical mathematical theories but also for many trivial reasonings of everyday life. Due to this basic use, most of the time implicite, the lack of education to paradoxes can be manipulated by a perverse Big Brother, in particular by blurring the distinction between A and not A (¬A) and in particular by fitting one into the other, for instance by inclusion. The naive set theory approach is poorly adapted for resisting to these awkward uses of circularity. Misleading is based on the following fact: to define an information flow s=(a, s), it is normally necessary to have defined s before setting s, which is impossible if s is nested in s'. There are alternative approaches for set theory to accept these new objects: for example, Forti & Honsell [FOR83]) Aczel [ACZ88]. Circularity phenomena can be modeled using sets if we are able to accept the sets are not well founded. We propose to introduce some elements, which makes it possible to make the link between circularity and the equally fundamental notion of fixed point. Therefore, the notions of convention and fixed point are linked together. In what follows, we restore the common thread which allows us to consistently understand the notion of circularity until arriving at the notion of fixed point.

5. Fix point[5] and circularity

Consider a binary relation R on a set S such that there does not exist an infinite sequence by b_0,b_1, …b_k… of elements of S such that $b_{n+1}\ R\ b_n$ for all n. If such a sequence exists the relation R is said to be badly-founded, and the infinite sequence which attests to it is called a descending sequence. The relation R is said to be circular if there exists a finite sequence b_0,b_1, …b_k such that: $b_0 = b_k$ and $b_{n+1}\ R\ b_n$ for n = 1,…, k,… Such a sequence is called a cycle. Otherwise R is said to be non-circular. By repeating the cycle infinitely, we see that if R is circular, it is badly-founded. In other words, if R is well-founded it is not circular. We can think, for example, of a computer program that enters a loop is a bad program. Examples: the set of natural integers ($\mathbb{N}$,<) is well-founded, while the set of relative integers ($\mathbb{Z}$,<) is not well-founded by construction because of the role of zero.

Set theory introduces the concept of set (denoted a in the following examples) and the membership relation denoted ∈. Classical set theory is based on the following fundamental Foundation axiom: the structure $\langle a,\in\rangle$ is well founded; We derive a principle of demonstration by ∈-induction. Let us indeed denote $\varphi(a)$ a set property and suppose that for all a, if $\varphi(b)$ for all $b \in a$, therefore thanks to the foundation principle for all a, we have $\varphi(a)$. If we remove the foundation axiom, we still have a principle of ∈-induction but this must be restricted to well-founded sets only. One of the important consequences of the foundation axiom is the following property: For all a, $a \notin a$. This leads to other consequences, such as: There

[5] In mathematics, and more precisely in algebraic topology, Brouwer's fixed point theorem is part of the large family of fix point theorems, which state that if a continuous function f satisfies certain properties, then there exists a point x_0 such that $f(x_0) = x_0$. The simplest form of Brouwer's theorem assumes that the function f is defined on a non-empty bounded closed interval I and with values in I. In a more general form, the function is defined on a compact convex K of a Euclidean and K-valued space.

does not exist a finite sequence $a_1,\ldots,a_n$ such that $a_1 \in a_2 \in \cdots \in a_n \in a_1$ Thus for all A there does not exist a non-empty set X such that $X = A \times X$ The unique solution of $X = X \times X$ is $X = \emptyset$

We can now approach circularity in formal computer sciences, which makes it possible to account for the algorithmic operations carried out by an electronic computer. For this part of the presentation, we refer to the Jon Barwise and Lawrence Moss Circles monograph [BAR96] – on the mathematics of non-well-founded phenomena. Let A be a set, a stream over A is an ordered pair $s = (a, s)$ where $a \in A$ and where s is another flow. From a flow we can carry out two elementary operations:

- Taking the first element which provides an element of A
- Taking the second element which provides a flow

Example $f(n) = (n, f(n + 1))$ is emblematic. For all n, $f(n)$ constitutes a flow where $f(0)$ is (0, (1, (2, (…)))) We cannot identify a flow with a set pair. Indeed, the constant flow denoted ca consists of the succession of a and ca itself, that is to say ca = (a, ca) which implies that ca is ill-founded. Let us denote the set of flows on A by A^∞; by definition $A^\infty \subseteq A \times A^\infty$and the converse is also true, thus $A^\infty = A \times A^\infty$. This writing highlights that the set $A^\infty$is the solution of a fixed-point problem.

To approach this kind of problem, we appeal to the lemma of the solution which we present succinctly. We start from a device model D which is placed in a state d and such that by applying a simple transition q we obtain the same state d; we can formally write: $d = \{(q, d)\}$, which we agree to rewrite more simply $d = \{q, d\}$, which can be reads like an equation. If we enlarge the universe of well-founded sets sufficiently, we hope to allow the "magnitude" $\{q, \{q, \{q, \{\ldots\}\}\}\}$ to become acceptable. It is therefore a question of being able to solve systems of equations: $x = \{q, x\}$ inducing $x = \{q, \{q, x\}\}$, which must still admit the same solution. The anti-foundation axiom states precisely that any system of equations of an arbitrary given form admits a solution[6]. On the other hand, such an equation can admit one or more solutions, or even an infinity. We will assume that every system admits a unique solution. To ensure this, it is useful to introduce a new concept, briefly described below named a flat systems of equations

A flat system of equations is a triplet $\mathscr{E} = \langle X, A, e\rangle$ formed from a set $X \subseteq \mathcal{U}$, a disjoint set A of X and a function e: $X \to \mathcal{P}(X \cup A)$, where X is the set of indeterminates data, A a set of atoms of $\mathscr{E}$. For all $v \in X$, the set $b_n = e_n \cap X$ is the set of indeterminates data on which v depends, cn = $e_n \cap A$ is the set of atoms on which v depends. A solution of $\mathscr{E}$ is a function s satisfying: $s_x = \{s_x: y \in b_x\} \cup c_x$ for all $x \in X$. The case $X = \emptyset$ is allowed. In this framework, it is then possible to build a new theory which replaces the classical theory of sets, characterized in particular by the foundation axiom recalled previously, by replacing it with a new axiom which is stated as an axiom of anti- foundation: *in this framework every flat system of equations admits a unique solution s.*

[6] The anti-foundation axiom (AFA) is an alternative axiom to the foundation axiom of set theory (i.e. any non-empty set x has a minimal element for membership on x, i.e. an element y having no element in common with x In the presence of the foundation axiom, we never have "x ∈ x") AFA proposes an extension of the set ontology It allows infinite descending chains for the membership relation on sets. For example, it allows a set to belong to itself or two distinct sets to belong to each other. In a universe of ZF theory (Zermelo Fraenkel without foundation axiom) it is always possible to define a part of it, (the von Neumann universe) which satisfies all the axioms of ZF and the foundation axiom, these are the well-founded sets. The consequence of the anti-foundation axiom is that von Neumann's universe is not the entire universe: there exist not well founded sets (sometimes called hyper-sets).

Let us note the set of solutions of the previous flat system: $Ens.\ Sol(\mathscr{E}) = \{sn: v \in X\} = s[X]$. Furthermore, let us note the collection of sets which are solution sets of a system admitting A as a set of atoms:

$$V_{\text{afa}}[A] = \cup\ \{Ens.\ Sol(\mathcal{E})\text{: } \mathcal{E}\text{-flat system with atome } A\}$$

Considering the equation $x = \{x\}$ as a flat system, we obtain the unique set which identifies with its own singleton. Obviously, $Ens.\ Sol(\mathscr{E})$ is transitive on sets: if b and c are sets such that $c \in b \in Ens.\ Sol(\mathscr{E})$, then $c \in Ens.\ Sol(\mathscr{E})$. If $\mathscr{E} = \langle X, A, e\rangle$ is such that atoms are primitive elements, and if s is a solution, then for all $x \in X$ $supp(s_x) \subseteq A$.

We can verify that Vafa$[A] = V[A]$, which means that it is always possible to find for a set such that $supp(a) \subseteq A$ there exists a flat system $\mathscr{E} = \langle X, A, e\rangle$ such that $a \in Ens.\ Sol(\mathscr{E})$. In particular Vafa $[\mathcal{U}] = V[\mathcal{U}]$. A reader who has some knowledge of category theory recognizes here a schematic adjunction. We introduce a partial order relation < by setting $x < y$ if and only if $y \in e_x$. The system is said to be well-founded if this relationship is well-founded. By applying Mostowski's collapse lemma, *we prove that the foundation axiom is equivalent to the statement that only well-founded flat systems admit solutions.*

As we mentioned before, all the flows must be solution of the equation $Z = A \times Z$. This equation admits at least one trivial, uninteresting solution: $Z = \emptyset$. In fact we are looking for the greatest solution. A^{∞} denotes the set of flows on A. For C any set or class, i.e. a function Γ: $C \rightarrow C$, c is called a fixed point if Γ(c) = c. In the class of all sets C by setting Γ(c) = A × c; thus A^{∞} must be the fixed point of Γ. The fundamental property that is exploited is the monotonic nature of the function Γ, which means: if $c \subseteq c'$ then $\Gamma(c) \subseteq \Gamma(c.)$. Let us then state the two classic fixed point theorems obtained by Knaster and Tarski:

Theorem of smallest fix point :

Every monotone operator Γ admits a smallest fixed point denoted $\Gamma *$ characterized by:
1) $\Gamma * = \bigcup_a \Gamma_a$ where Γ_a is defined recursively on the ordinals by:
$\Gamma_a = \bigcup_{b<a} \Gamma(\Gamma_b)$
2) $\Gamma *$ is the smallest closed class for Γ

Theorem of greatest fix point :

Every monotone operator Γ admits a largest fixed point denoted Γ^* characterized by:
1) Γ^* =∪ $\{a: a\ est\ un\ ensemble\ \Gamma-$ correct$\}$
2) $\Gamma^*$is the largest Γ-correct class
A set, or a class, G is called Γ-correct if $G \subseteq \Gamma(G)$; it is qualified as Γ-closed if $\Gamma(G) \subseteq G$,
G is a fixed point if it is at both closed and correct.

We previously highlighted how such circularity phenomena present a universal character whatever the field of study considered. Thanks to these two theorems, the fixed point problem admits at least one solution. Given the theoretical approach , here briefly introduced, circular processes admit a formal solution in very general situations.

6. Back to foundation axiom

In this additional paragraph, we return to the foundation axiom. As we have highlighted previously, it is essential to understand its meaning. It is a source of perplexity for anyone approaching mathematics formally for the first time. This axiom is precisely called foundation axiom because it is the pilar of set theory; its impact is decisive for the entire mathematical structure. It is worth noting an astonishing phenomenon with intuitive natural numbers: let m and n be two natural numbers with $m < n$ we then have two statements that we must distinguish that of belonging and that of inclusion: $m \in n$ and $m \subset n$. Classical set theory excludes the possibility $x \in x$ and in the previous case we actually come close to the precipice: if we identify n with $\{1, 2,\ldots, n\}$ we arrive at the result $n \in n$. This justifies the fact that the integer n is identified with the set $\{0,1, 2,\ldots, n\text{-}1\}$, an identity whose generalization leads to the theory of ordinals (this is generally confusing for beginners). To evacuate this situation from a statement: $\exists x(x \in x)$, the creators of the modern theory of mathematics, after intense discussions (the famous crisis of mathematics at the very end of the 19th century), agreed to introduce the following decisive foundation axiom:

For any non-empty set a, there exists an element $b \in a$ such that the intersection of sets b and a is empty $b \cap a = \emptyset$; namely:

$$\forall x(x \neq \emptyset \Rightarrow \exists y(y \in x \; et \; y \cap x = \emptyset))$$

Intuitively, the underlying justification consists of stating that a set is composed of elements "of the same nature". This suggests that there is a hierarchy between the sets. Given a non-empty set c, we form $a = c \cup \{c\}$; c appears both as a subset of a and as an element of a:

- The statement $c \subset a$ shows a and c as elements of the same set $\mathcal{P}(a)$
- The statement $c \in a$ considers a hierarchy between c and a

Therefore $c \cap a = c \neq \emptyset$, then $c \in a$ et $c \cap a \neq \emptyset$, If we deny the foundation axiom we obtain the statement:

$$\exists x(x \neq \emptyset \; et \; \forall y(y \in x \Rightarrow y \cap x \neq \emptyset))$$

This statement then means that there exists a non-empty set a such that for any element $b \in a$, $b \cap a$ would be non-empty. For any element b of a, the hierarchy demonstrated by the statement $b \in a$ is then negated by the statement $b \cap a \neq \emptyset$, *thus the union operation allows us to consider the case $a = c \cup \{c\}$ where this hierarchy between elements of the same set and the set itself is disturbed. The foundation axiom guarantees that this hierarchy cannot be totally annihilated.*

The observations that we have just recalled show that *there is a link between the axiom of foundation and hierarchy. Therefore, the introduction of the anti-foundation axiom is a way of shaking up the existence of such a hierarchy*. Should we deduce that the emergence of a convention constitutes a way of calling into question an existing hierarchy? here's one sociological and anthropological question of a philosophical nature that we propose to our reader before temporarily concluding this socio-physical reflection.

7. Conclusion

Recalling the original and fruitful study of the analytical philosopher D. K. Lewis, we have woven the links between the concept of convention which finds its origins in the human sciences, from law to philosophy via sociology, and the concept of set which is the first gearwheel of mathematics, logic included. These concepts and the related issues lead to very operational diagrams that we find traces for example in information technologies and computer sciences. Mathematical analysis makes it possible to understand the sources of blurring and shaking up hierarchies that conspire theories and the commercialized assurance of opinion influencers are based on. In all cases it is the capability of designing an universality of reasoning which is called into question as well as the existence of a rational fixed point. Based on manipulated cognitive faults, confusions here explained, give birth to a relativism whose deadly and evil nature everyone can appreciate today. Obviously, the social network contribute to a mystification always based on badly found mathematical set approach.

It is quite interesting to note that in the same week when P. Riot orally presented the content of this note in Roquebrunes Scientific Meeting , a short work written by Professor Mario Barra-Jover was publiched, entitled “On regularity”, [BAJ23] in which the author also exploits the work of D. K. Lewis, coupling it with that of Donald Davidson, -another major analytical philosopher-, stating that everything that is general, from physical laws to ethical principles, would fall under normative circular statements imposing on us a way of looking at the world in order to act effectively in it. The rest of the regularities, starting with those shaped by our linguistic knowledge, would therefore be the result of spontaneous processes affecting the individual or the collective. From this brief summary we see obvious correlations with the notion of convention which served as our starting point. Its duality would be, a major characteristic of human behavior for facing the chaos of the world.

In memoriam : Dr. Pr. Anca Gheorghiu

As Pr. Fedor Kusmartsev writes in his perfect English of Loughborough University :

“Anca was an extraordinary individual, and her loss is deeply felt by all who had the privilege of knowing her. Anca's impact on the scientific community, particularly in the field of econo-physics, is immeasurable. Her dedication, passion, and intellect were evident in every aspect of her work. As a scientist, she illuminated the path forward with her innovative research and unwavering commitment to advancing knowledge. Beyond her professional achievements, Anca was a remarkable person (...) a source of inspiration to many. Her kindness, warmth, and generosity touched the lives of everyone around her. She had a rare ability to bring people together and foster a sense of community wherever she went. Anca's presence was like the sun, radiating light and vitality to all who crossed her path. Her spirit and enthusiasm were contagious, uplifting those fortunate enough to know her. Though she may no longer be with us in person, her legacy will continue to shine brightly through the countless lives she touched and the profound impact she made on the world. Her ideas and initiative will stay forever as perpetum fire.”.

The french authors of this note share Fedor's feeling and compassion for Anca relatives and friends.

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