A DYNAMIC MACRO-MODEL ACCORDING TO NATIONAL ACCOUNTS

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Abstract. *The 2008 economic crisis was not forecastable by existing macroeconomic models. Thus macroeconomics needs new tools. We introduce a dynamic macro model coincident with National Accounts that reports the money flows between macroeconomic sectors. The monetary linkages between macroeconomic sectors create fluctuation sin to macroeconomic variables that cannot be explained by General Equilibrium type of models. We show that the connections between Households' and Non-Financial Firms' Sector together with price, wage, and interest rate adjustment create cyclical behavior in production, employment, consumption, and investment. We study also the stability of macroeconomic systems. Our study serves as a basis for systems analytic macro-models that can model positive and negative feedbacks in production systems together with interactions between real and financial markets.* ***JEL*** *E01, E10.*

Keywords: *Macroeconomic dynamics, National Accounts, Interactions between macroeconomic sectors.*

1. Introduction

The economic crisis at 2008 could not be forecasted by existing macroeconomic models. Kobayashi [1] writes: "In fact, the crisis we are currently experiencing may call for a change in the theoretical structure of macroeconomics. In my view, a macroeconomic approach that encompasses financial intermediaries and places them at the center of its models is necessary. We need a new paradigm of economic thought".

Leijonhuvud [2] criticizes the General Equilibrium (GE) framework: "The economy is a large complex dynamical system which is in large measure self-regulating. Its self-regulatory features are the negative feedback loops that we refer to as 'market mechanisms': excess demand for a good raises its price which in turn reduces its excess demand. … If

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the economy is displaced not too far from equilibrium, market forces will bring it back; if displaced too far, they will be ineffective or may work perversely. … Theories that assume that the economy is a stable general equilibrium system … do not hold in general. The instabilities that such theories ignore are precisely those problems that should be the particular responsibility of macroeconomists."

Caballero [3] stresses the interactions in macro-models: "The recent financial crisis has damaged the reputation of macroeconomics, largely for its inability to predict the impending financial and economic crisis. … up to now the insight-building mode of the periphery of macroeconomics has proven to be more useful than the macro-machine-building mode. … the periphery gave us frameworks to understand phenomena such as speculative bubbles, leverage cycles, fire sales, liquidity runs … we need to spend much more effort in understanding the topology of interactions in real economies."

Roncaglia [4] suggests that macroeconomic balance sheets are important elements in future macroeconomic models: "The economic crisis we are now experiencing may entail radical changes in … macroeconomics … the series of events leading to the present crisis has made it quite clear that there are no automatic rebalancing mechanisms: thus analysis of the macroeconomic balance sheets and their interactions constitutes an important element in reconstruction of macroeconomics. In itself, the analysis of macroeconomic balance sheets is compatible both with the mainstream theories and with the heterodox theories: the context within which it is applied determines the cause-and-effect links characterizing the various interpretations of the economic events… Much still remains to be done, but the foundations for new macroeconomics are already available".

We agree with these authors. Our model corresponds to the principles in System of National Accounts (SNA) [5] and thus we contribute in macroeconomic modelling that is in exact correspondence with SNA. The study is organized as follows. First, we describe the principles of National Accounting and compare them with the principles of modelling flow systems in Physics and Engineering. We show that in modelling money flows, identical techniques can be applied as in Physics in modelling flow systems. Adjustment equations for the analyzed macro-level money flows as well as for price, wage, and interest rate are then defined, and by the constructed model we show how the mutual connections between households' and non-financial firms'sectors create different kind of dynamic behavior in product, labor, and capital markets.

2. National Accounts as a Money Flow System

The analogy between a money and a water flow system is demonstrated in Figure 1. The difference between inflow and outflow of water (money) during time Δt corresponds to the Profit and Loss Statement of a firm, and the amount of water (money) in the vessel corresponds to the net wealth of the firm in its Balance Sheet.

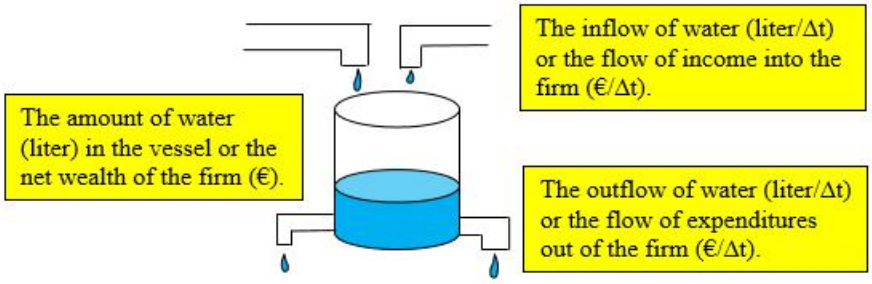


Figure 1. The stock-flow principles in the measurement of a water (money) flow system.

The SNA and a money flow diagram are two ways to describe the real and the money flows between macro-economic sectors. In every trade, the expenditures of the buyer equal the gross revenues of the seller of which sales and product taxes are delivered to the state. Every bought good causes a money flow (its payment) in one direction, and a real flow (its delivery) in the opposite direction. The production process of an economy can thus be modelled on the basis of the money or the real flow system that have parallel events in both processes.

The nominal GDP of an economy is obtained by adding the value-added of all firms in the economy in a time unit at current prices. By the nominal GDP we can approximate the real GDP − the aggregate flow of production in the economy − as follows. Let all production volumes be measured in mass units, kg, and let the unit price of the product of firm i be denoted by $p_i\ (€/kg)$ and the real value-added of the firm by $q_i(kg/\Delta t)$, where Δt is a time unit. Let n one-product firms exist in the economy. Then

$$GDP = \sum_{i=1}^{n} p_i q_i \approx \bar{p} \sum_{i=1}^{n} q_i \quad \Rightarrow \quad \frac{GDP}{\bar{p}} \approx \sum_{i=1}^{n} q_i ,$$

where $\bar{p}\ (€/kg)$ is the average of $p_i,\ i = 1, \dots, n$. An estimate for the real GDP, $GDP_R = \sum_i q_i(kg/\Delta t)$, is thus obtained by dividing the nominal GDP by $\bar{p}$.

2.1. Modelling an Account

We have three analogous ways to present an account: 1) T-form, 2) Equation -form, and 3) Block diagram -form. These are shown in Figure 2 where R is revenues, C costs, and B the balancing item that makes both sides of the accountequal; all these quantities have unit $€/\Delta t$. Block diagrams are common in physics and engineering in presenting flow systems, and we show here that they can be used in money flow systems too.

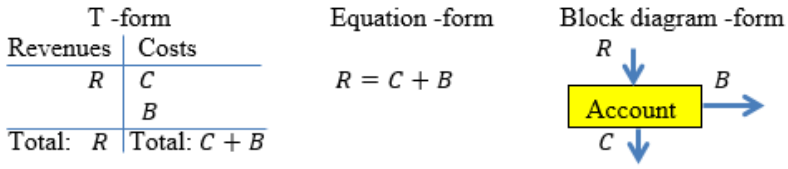


Figure 2. Three ways of presenting an account.

Table 1.
Accounting (transaction) matrix between macroeconomic sectors

	Households (HS)	Firms (FS)	Government (GS)	Sum
1. Consumption	$-C$	C		0
2. Government expenditures		G	$-G$	0
3. Output		$[Y]$		
4. Factor income	W	$-W$		0
5. Taxes	$-T$		T	0
6. Change in money stock	$-\Delta H$		ΔH	0
Sum	0	0	0	0

The transactions matrix between macroeconomic sectors in Table 1 is taken from Godley & Lavoie [6 p. 60]. This corresponds to the following money flow diagram between the sectors.

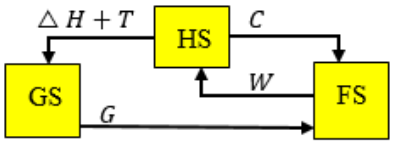


Figure 3. The money flow diagram corresponding to Table 1.

The last column on zeros in Table 1 is seen in Figure 3 so that every money flow (e.g. C) leaving an account (HS) ends into another account

(FS). This rule holds for every money flow. The last row of zeros in Table 1 is seen in Figure 3 so that the sum of incoming money flows in every sector equals with the sum of leaving money flows. The corresponding *T*-accounts and system of equations below verify these adding up rules in Table 1.

HS		FS		GS		
W	$C+T+\Delta H$	$C+G$	W	$T+\Delta H$	G	HS: $W = C + T + \triangle H$,
						FS: $C + G = W$,
						GS: $T + \triangle H = G$.

Thus these four ways of presenting the system correspond to each other exactly. However, in Table 1 is assumed that the balancing item in every account is zero. This hindering restriction is released in the following which is critical for understanding the stock-flow connections between the accounts. If all flow accounts balance, the corresponding stocks stay constant which situation is rare in the real world.

3. National Accounts of Non-Financial Firms'and Households' Sectors

In this study we concentrate on money flows between Households' Sector (HS) and Non-Financial Firms' Sector (NFS). For this reason, we omit from the analysis international connections (exports and imports), public sector's role (public expenditures, taxes, social contributions etc.), the acquisition of non-financial non-produced assets, and financial accounts i.e. lending and borrowingand acquiring or issuing of financial assets and liabilities. We show that our model can be extended by all these elements, but every item will complicate the model. Thus, we start with a simplified model that can be extended later.

3.1. Non-Financial Firms' Sector

NFS is obtained by aggregating the National Accounts of every non-financial firm in a country so that the mutual money flows between non-financial firms cancel. The opening balance sheet of NFS is

Assets (€) Liabilities and net wealth (€)

A_{i0}	$\Psi_{i0} + NW_{i0}$

Figure 4. The opening balance sheet of NFS.

where $NW_{N0} = A_{N0} - \Psi_{N0}$ is the net wealth of NFS as the difference between its assets A_{N0} and liabilities Ψ_{N0}. In the first flow account of NFS – the production account– intermediate consumption is subtracted from the value of output ($\in P1$) (official abbreviations of the items are in parentheses). This gives the value added of NFS. The gross value added ($\in B1G$) of NFS is $GVA_N = value\ of\ output_N - H_N$, where $H_N (\in P2)$ is the intermediate consumption of NFS on the products of domestic firms.

The Net Value Added (*NVA*) ($\in B1N$) of NFS is obtained by subtracting the consumption of fixed capital $P_N (\in K1)$ from GVA_N, $NVA_N = GVA_N - P_N$. In SNA, the consumption of fixed capital is not made on the basis of firms' depreciation bookings but by the estimated consumption of firms' capital goods. In SNA, valuation is made in gross and in net terms and we apply here net valuation that contains the consumption of capital too. Two kind of pricing principles are applied in SNA. If production is valued at basic prices, taxes and subsidies on products are not included, and if production is valued at producers' prices, taxes and subsidies on products are included in prices. Here we value production at producers' prices.

Current transfers consist of four items: 1) Net current taxes on income, wealth etc. ($T_I \in D5$), 2) Social contributions and benefits other than social transfers in kind ($B_S \in D61 + D62$,), 3) Other current transfers ($N \in D7$), and 4) Capital transfers ($B_C \in D9$). In a transfer, an institutional unit provides a good, a service, or an asset to another unit without receiving from the latter any direct counterpart ([5], 161). Social benefits can be divided into pensions and other social benefits. Other social benefits are collected either voluntarily or obligatory from households or firms, and are paid as transfers to the beneficiaries. Social transfers in kind contain e.g. final consumption expenditures undertaken by government on behalf of households (ibid, 167-178). In the following, we assume the time unit to be one$week$ even though National Accounts are officially reported quarterly. Thus, NFS faces the following money flows with unit €/$week$.

Revenues: Households' ($C_{1N} \in P31$) and government's ($C_{2N} \in P31$) individual final consumption expenditureson the products of NFS, $C_N = C_{1N} + C_{2N}$, Government's collective final consumption expenditures ($G_N \in P32$) on the products of NFS, Firms' gross fixed capital formation ($I_N \in P5$) on the products of NFS, Sales of intermediate goods to domestic firms (Z_N), Exports of goods and services ($X_N \in P6$) of NFS, Property income ($PI_{RN} \in D4$), Subsidies on production ($B_{PN} = B_{P1N} + B_{P2N}$, $B_{PN} \in$ D3, $B_{P1N} \in D31$, $B_{P2N} \in D39$), Social contributions and benefits other than social transfers in kind (B_{SNR}), Other current transfers (N_{NR}),

and Capital transfers (B_{CNR}) received by NFS. Z_N is an auxiliary variable, sub-index R refers to revenues, $D3$ is Subsidies, $D31$ Subsidies on products, and $D39$ Other subsidies on production.

Costs: Expenditures on intermediate products (H_N), Employers' social contributions ($T_{SN} \in D12$), Taxes on production and imports ($T_{PN} = T_{P1N} + T_{P2N}$, $T_{PN} \in D2$, $T_{P1N} \in D21$, $T_{P2N} \in D29$), Gross wages and salaries ($W_N \in D11$), Current taxes on income, wealth etc. (T_{IN}), Own gross investment in fixed capital and changes in inventories (I_{ON}), Social contributions and benefits other than social transfers in kind (B_{SNP}), Other current transfers (N_{NP}), Capital transfers (B_{CNP}), Imports of goods and services ($M_N \in P7$), Net acquisition of non-financial non-produced assets ($N_{ANn} \in K2$), and Property costs (PI_{NP}). Sub-index P refers to payments, $D2$ to Taxes on production and imports, $D21$ to Taxes on products, and $D29$ to Other taxes on production.

Every firm does not face all these money flows, however. For example, if firm *i* does not produce investment goods, then $I_i = 0$. In SNA in imports and exports, no distinction is made between consumption and investment goods, and between final and intermediate goods. We assume that all imports are carried out by the firms in the home country that sell the goods forward to domestic customers. Taxes on production and import containe.g. value added tax. The imported goods sold by NFS are subtracted from the market value of output to get the value of domestic production. In the Generation of Income Account (GIA), the primary income components paid to the corresponding units are subtracted from the net value added. *Primary incomes* accrue to economic units due to their involvement in processes of production or ownership of assets that may be needed for production. This may contain compensation of employees, taxes on production and imports less subsidies, operating surplus or mixed income, and property income ([5], 131).

We showed in Figure 2 that an account can be presented as a block diagram. In Figure 5 is the block diagram of the sequence of Generation of Income Account GIA, Allocation of Primary Income Account (APIA), and Secondary Distribution of Income Account (SDIA) of NFS. Notice that money is not stored in any of these accounts because all incoming money in these accounts flows out in the form of expenditures and balancing items. Assets and liabilities are "stored" the opening and closing balance sheets, see Figure 4.

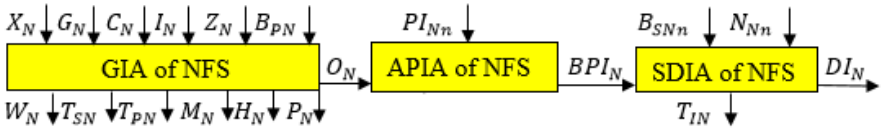


Figure 5. The GIA, APIA, and SDIA of Non-Financial Firms' Sector.

The balancing item in the GIA is the operating surplus plus mixed income, $O_N \in B13N$, that is transferred to the APIA. *Mixed income* is the balancing item of unincorporated enterprises in the households' sector that corresponds to remuneration for work carried out by the owner or members of his family including profits gained as entrepreneur ([5], 132). In the APIA, O_N is adjusted by property incomes PI_{NR} and costs $PI_{NP}, PI_{Nn} = PI_{NR} - PI_{NP}$ (through this study sub-index n refers to net). The balancing item of APIA is the balance of primary income $BPI_N \in B5NT$. In the SDIA, BPI_N is adjusted by current transfers ([5], 159). This gives the disposable income of NFS, $DI_N \in B6N$, see Figure 5. The equations of the accounts are:

$$O_i = X_i + G_i + C_i + Z_i + I_i + B_{Pi} - H_i - W_i - T_{Si} - T_{Pi} - M_i - P_i, \quad (1)$$

$$BPI_i = O_i + PI_{iR} - PI_{iP}, \quad DI_i = BPI_i + B_{SiR} + N_{Ri} - B_{SiP} - N_{Pi} - T_{Ii}. \quad (2)$$

Because firms do not participate in final consumption, the whole disposable income of NFS is saved, $DI_N = S_N \in B8N$. Savinglinks the Use of Disposable Income Account (UDIA) and the Capital Account (CA) of NFS, and here we omit the item *Adjustment for the change in pension entitlements* (= $D8$), see [5], 181). The reason for this is that pension contributions are included in social contributions (ibid. 168).

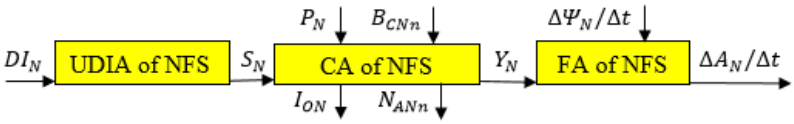


Figure 6. The UDIA, CA, and FA of Non-Financial Firms' Sector.

The UDIA, the CA, and the financial account (FA) of NFS are shown in Figure 6. The flow of consumption of capital P_N into CA can be explained in two ways: 1) CA measures the capital stock in net terms, and because investments increase the stock, the consumption of capital must be on the other side of the account. Thus, the capital stock of NFS accumulates via firms' net investment, $I_{ON} - P_N$. Actually, firms do not pay anything of the consumption of their capital stock, and because

P_N enters as an "expenditure" item in the GIA of NFS, it must enter as a "revenue" item in some other account (CA) so that in the consolidation of the accounts of NFS, P_N cancels. 2) The higher P_N is, the smaller is O_N, and the less the firm pays current taxes on income that are calculated on the basis of O_N. Thus, P_N increases the savings of NFS and so NFS benefits of its bookings of consumption of fixed capital even though this consumption speeds up the replacement of capital goods.

The net capital transfers received by NFS are denoted by $B_{CNn} = B_{CNR} - B_{CNP}$, I_{ON} is the (own) gross investment including changes in inventories, $Y_N (\in B9T)$ the net lending/borrowing, $\Delta\Psi_N/\Delta t$ the net issuing of financial liabilities, and $\Delta A_N/\Delta t$ the net acquiring of financial assets of NFS in the week; the last two are asset flows with $\Delta t = 1\, week$. N_{ANn} is the net acquiring of non-financial non-produced assets, and if $N_{ANn} < 0$, NFS has earned money by selling more than buying these assets.

If the net lending/borrowing Y_N is positive, NFS has a surplus after its investments in fixed capital and the sector can invest this money in financial assets or repay its loans. However, if $Y_N < 0$, this deficit must be financed by borrowing or issuing liabilities (notice that $\frac{\Delta A_N}{\Delta t}$ and $\frac{\Delta \Psi_N}{\Delta t}$ may be negative too). The acquiring of financial assets and liabilities is not modelled in this study to keep the model tractable, however, but we show how these items can be included in the model. Now, FA is the last flow account of NFS and changes in stocks ΔA_N, $\Delta\Psi_N$ with unit € are added, respectively, in the initial assets (A_{N0}) and liabilities (Ψ_{N0}) in the opening balance sheet of NFS. This gives the closing balance sheet of NFS at the end of the week. We omit here the *Other changes in the volume of assets account* and the *Revaluation account* that correct the amounts and the values of assets during the year due to non-intentional events, see [5], 237-255.

3.2. Households' Sector

The Non-Profit Institutions Serving Households (NPISHs) – e.g. political parties and religious communities – are consolidated into the Households' sector (HS) because these units behave like households. HS contains wage earners and owner-occupied firms owned by households, which explains why the GIA of HS contains similar items as that of NFS. The National Accounts of HS are mainly identical to those of NFS but the revenues as a producer are in a minor role as compared with labor income.

Another difference is that the disposable income DI_H of HS is divided into consumption C_1 and saving S_H while firms do not participate in final consumption. In the GIA, $C_H = C_{1H} + C_{2H}$ and HS pays gross wages W_{HP} to its laborers. In the APIA, HS gets gross wages from the other sectors W_T and so the gross wage income of HS is $W_{HR} = W_{HP} + W_T$ while NFS does not receive wage income. The accounts of HS+NPISH (shortly HS) are in Figure 7, 8.

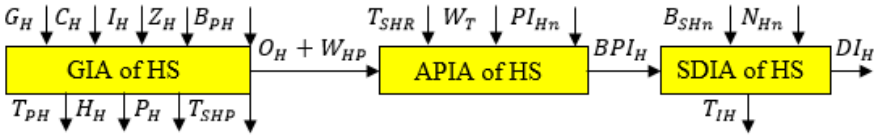


Figure 7. The GIA, the APIA, and the SDIA of HS.

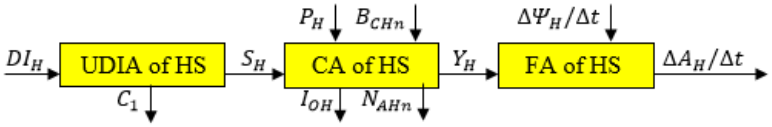


Figure 8. The UDIA, the CA, and the FA of HS.

Other items in these accounts are the same as those in NFS. In the following we do not consider the role of HS as a producer and so we cancel the GIA from HS. Thus, we set $O_H = W_{HP} = P_H = 0$. The reason for setting $P_H = 0$ is that the capital stock of HS consists solely of those of owner occupied firms that are omitted here.

4. Modelling Dynamic Macro-Level Behavior

Moving terms into different sides of Eq. (1) we can write it as

$$C_N + G_N + I_N + (Z_N - H_N) + (X_N - M_N) = W_N + O_N + T_{SN} + T_{PN} + P_N - B_{PN}. \quad (3)$$

Now, at the level of NFS the expenditures on and revenues from domestic intermediate goods are equal. Thus, $Z_N = H_N$ and so these terms cancel from Eq. (3). Then on the left hand side of Eq. (3) is that part of GDP of the country created by NFS calculated on the basis of expenditures, and on the right hand side is that part of GDP of the country created by NFS calculated on the basis of income. The GIA of NFS thus creates the GDP equation according to the principles in SNA. If we would model the behaviors of Government Sector (GS), Households' Sector (HS), Financial Firms' Sector (FS), and Non-Profit Institutions Serving

Households (NPISH) Sector as accurately as that of NFS, we would obtain the GDP of the whole economy by consolidating the GIA's of all these sectors. However, here we simplify the analysis by eliminating international trade, public sector, financial firms, and HS as a producer from the analysis, and thus our GDP measures are constructed on the basis of NFS alone. Thus, we cancel items $G_N, X_N, M_N, T_{SN}, T_{PN}, B_{PN}$ from Eq. (3) and in the following we simplify the modelling also by omitting the consumption of capital, i.e. $P_N = 0$. The GDP equation is then

$$C_N + I_N = W_N + O_N.$$

The mutual money flows between NFS and HS are shownin Figure 9 where we have consolidated the accounts as $GASUA = GIA + APIA + SDIA + UDIA$. Here we omit government transactions and the acquisition of non-financial non-produced assets, and so $B_{SNn} = N_{Nn} = T_{IN} = 0$.

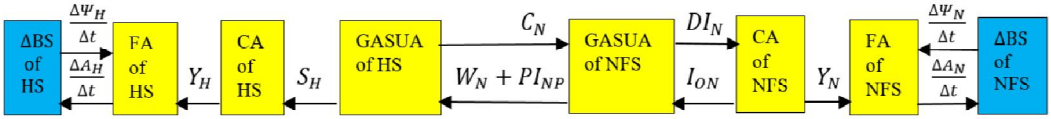


Figure 9. The money flows between NFS and HS.

The Changes in Balance Sheets (ΔBS) accounts show how money flows change the "stocks" of net wealth of the sectors, and how the acquired financial assets and liabilities affect the liquidity of the sectors. This diagram reveals the stock-flow principles in money flow calculations introduced in Figure 1. In the following we omit the asset flows, however, to keep the modelling tractable. In the consolidation of $GASUA$, the balancing items of the consolidated accounts vanish. The equations of accounts SDIA and UDIA in HS e.g. are

$$BPI_H + B_{SHn} + N_{Hn} = T_{IH} + DI_H, \quad DI_H = C_1 + S_H.$$

Adding the left and the right-hand sides of these equations, we get the equation of account SDIA+UDIA as

$$BPI_H + B_{SHn} + N_{Hn} + DI_H = T_{IH} + DI_H + C_1 + S_H \quad \Leftrightarrow \quad BPI_H + B_{SHn} + N_{Hn} = C_1 + S_H.$$

Thus, in the latter form of the equation, balancing item DI_H does not exist. Because the only sector taking part in final consumption is HS, then $C_1 = C_N$. The only property income in the model is the dividend and interest earnings NFS pays to HS, and thus $PI_{HR} = PI_{NP}$. Notice that if S_H, DI_N were negative, the sectors must finance their deficits ($Y_H, Y_N < 0$) by issuing liabilities i.e. making $\Delta\Psi_i/\Delta t > 0$, $i = H, N$.

4.1. Adjustment in Labor, Capital, and Production

We assume the following aggregate level production function for NFS

$$Q(L(t),K(t)) = A_L L(t) - (B_L/2)\,L^2(t) + A_K K(t) - (B_K/2)K^2(t) + HL(t)K(t), \qquad (4)$$

where $Q\ (kg/week)$ is the flow of production created by NFS, $L(t)\left(\frac{h}{week}\right)$ the weekly hours of labor NFS uses in production, $K(t)(€)$ the monetary value of the capital stock of NFS, and A_L, B_L, A_K, B_K, H with units kg/h, $kg \times week/h^2$, $kg/(week \times €)$, $kg/(week \times €^2)$, $kg/(h \times €)$, respectively, are constants that make function (4) well-defined in measurement units[1]. The marginal productivities of labor and capital are

$$\frac{\partial Q}{\partial L} = A_L - B_L L + HK, \qquad \frac{\partial Q}{\partial K} = A_K - B_K K + HL,$$

with units kg/h and $kg/(€ \times week)$, respectively. Positive constant H represents the connection between labor and capital in the production processesin NFS so that increasing either production factor raises the productivity of the other factor. The weekly labor costs of NFS (= the weekly labor income of HS) are $W_N = wL$, where $w\ (€/h)$ is average hourly wage oflabor, and the weekly costs of capital are $PI_{NP} = rK$, where $r(1/week)$ is the average weekly interest (dividend) rate NFS pays to the people in HS whoown the firms.

Because increases in inventories of produced goods is a part of investment, the weekly production is equal with consumption and investment, $pQ = C_N + I_N$, where $p\ (€/kg)$ is the average unit price of produced goods. Thus, NFS produces consumer goods to HS and investment goods to itself (some firms in NFS produce investment goods and others buy them). According to microeconomic theory, the profit function of NFS is

$$\Pi_N(t) = pQ(L(t),K(t)) - wL(t) - rK(t) \;\Leftrightarrow\; pQ\big(L(t),K(t)\big) = \Pi_N(t) + rK(t) + wL(t), \qquad (5)$$

[1] Measurement units are in parentheses after the quantities, see [7]. The volume of production is measured in units kg because various units for different products would complicate the aggregation of production volumes. The measurement unit of interest rate (or rate of return) $1/week$ results from dividing a weekly flow of money with unit $€/week$ by that of invested capital, €.

where $\Pi_N(t) + rK(t) = O_N$ is the operating surplus plus mixed income of NFS that contains property incomes $rK(t) = PI_{NP}$ too NFS pays to HS. The latter form of Eq. (5) is the money flow equation between NFS and HS so that $C_N + I_N = pQ = \Pi_N(t) + rK(t) + wL(t) = O_N + W_N$ in Figure 9 (notice that $O_N = DI_N + PI_{NP}$ and $I_N = I_{ON}$ because NFS is the only sector producing and buying capital goods. The microeconomic profit concept of NFS thus equals with macroeconomic disposable income concept, $\Pi_N(t) = DI_N = S_N$.

We can now question, what causes the dynamics of employment, capital stock, and GDP in this model economy? In neoclassical microeconomic theory, firms are assumed to maximize their profit. We assume here, too, that NFS behaves in a profit-seeking way. Taking the time derivative of the first form of Eq. (5) (notice that NFS is assumed to be price taker in product, labor, and capital goods' markets) we get

$$\Pi'_N(t) = \left(p\frac{\partial Q}{\partial L} - w\right)L'(t) + \left(p\frac{\partial Q}{\partial K} - r\right)K'(t). \quad (6)$$

Now, NFS likes to increase its profitability with time i.e. to get $\Pi'_N(t) > 0$. The following adjustment rules for labor and capital inputs increase the profitability of NFS with time (see [8], 277, 318):

$$L'(t) > 0 \; if \; p(\partial Q/\partial L) - w > 0, \; L'(t) < 0 \; if \; p(\partial Q/\partial L) - w < 0, \; L'(t) = 0 \; if \; p(\partial Q/\partial L) - w = 0,$$
$$K'(t) > 0 \; if \; p(\partial Q/\partial K) - r > 0, \; K'(t) < 0 \; if \; p(\partial Q/\partial K) - r < 0, \; K'(t) = 0 \; if \; p(K) - r = 0.$$

We can thus write the following adjustment equations for labor and capital that fulfill the willingness of firms in NFS to adjust their employment and capital stock with time so that their profit increases

$$m_L L'(t) = p(\partial Q/\partial L) - w, \qquad m_K K'(t) = p(\partial Q/\partial K) - r; \quad (7)$$

constants m_L, m_K with units $€ \times week^2/h^2, 1/€$, respectively, measure the inertia of labor force and capital stock that correspond to inertial "masses" in Newtonian mechanics. Then $p(\partial Q/\partial L) - w$, $p(\partial Q/\partial K) - r$ can be interpreted as the "forces" that cause the acceleration in employment $L'(t)$ with unit $h/week^2$ and the flow ofthe capital stock i.e. the investment $K'(t) = I_{ON}$ of NFS with unit $€/week$. The equilibrium state of the system $L'(t) = K'(t) = 0 \Leftrightarrow p(\partial Q/\partial L) = w$, $p(\partial Q/\partial K) = r$ corresponds to the maximum profitability of NFS in a stable case, and the minimum profitability of NFS in an unstable case.

4.2. Adjustment in Consumption

In consumption, we assume that households adjust their consumption to increase their welfare. The following target function (see [9]) is assumed for HS

$$R = \eta(wL)U(X) - pX,$$

where $U(X)$, $U'(x) > 0, U''(x) < 0$ with unit $util/week$ is the weekly utility function of HS that depends on the real flow of consumption X $(kg/week)$ of HS. Factor $\eta(wL), \eta'(wL) > 0$ with unit $€/util$ transformsutility into monetary form so that $\eta(wL)U(X)(€/week)$ is the monetary valuation of consumption X of HS (notice that we omit the effect of property income $rK(t)$ on consumption X due to its uncertain nature). The costs of consumption are $C_N = pX$ $(€/week)$ and so R measures the monetary value of consumption $X$for HS in net terms. To demonstrate the solutions of the system, we assume a specific function for utility $U(X) = aX - (b/2)X^2$, where a, b are constants with units $util/kg$and $util \times week/kg^2$, respectively. Further, we assume $\eta(wL) = cwL$, where c is a positive constant with unit $week^2/(util \times kg)$. These units make the functions well-defined. Now, HS aims to increase its utility with time and so it adjusts consumption X as

$$m_X X'(t) = cwL(t)\big(a - bX(t)\big) - p(t), \tag{8}$$

where positive constant m_X with unit $€ \times week^2/kg^2$ represents the inertia in consumption, $X'(t)$ with unit $kg/week^2$ is the acceleration of consumption, and $cwL(t)\big(a - bX(t)\big) - p(t)$ can be interpreted as the "force" causing this acceleration, see [9]. Thus, HS increases consumption $(X'(t) > 0)$ if its marginal willingness to pay for one kilogram of goods $cwL(t)\big(a - bX(t)\big)$with unit $€/kg$ is higher than price $p(t)$, and vice versa. In the equilibrium situation $X'(t) = 0$ the following holds: $cwL(t)\big(a - bX(t)\big) = p(t)$.

4.3. Adjustment, Wage, and Interest Rate

We assume the following adjustment equations for price, wage, and interest rate:

$$p'(t) = k\big(X(t) - Q(t)\big), \tag{9}$$

$$w'(t) = s[p(t)(A_L - B_L L(t) + HK(t)) - w(t)], \tag{10}$$

$$r'(t) = v\big[p(t)\big(A_K - B_K K(t) + HL(t)\big) - r(t)\big], \tag{11}$$

where $Q(t) = A_L L(t) - (B_L/2)\,L^2(t) + A_K K(t) - (B_K/2)K^2(t) + HL(t)K(t)$ and k, s, v are positive constants with units $€/kg^2$, $1/week$, $1/week$, respectively. These correspond to spring constant k in Hooke's law$F = k\Delta x$ in Physics, where F is the force and Δx the deviation of the body from its rest position. The adjustment in price takes place according to excess demand in consumption over productionin a perfectly competed market (thus $X(t) - Q(t)$ corresponds to Δx in Hooke's law), see [10]. Wage dynamics occurs due to the difference in the monetary value of marginal productivity of labor, $\partial\Pi/\partial L = p(A_L - B_L L(t) + HK(t))$ and wage $w(t)$. Wage increases $(w'(t) > 0)$ if $p(A - BL(t) + HK(t)) > w(t)$ and vice versa, because employees can bargain wage increases if the value of marginal productivity of labor exceeds wage (term $p(A - BL(t) + HK(t)) - w(t)$ corresponds to Δx in Hooke's law). The adjustment in interest rate is constructed similarly as wage adjustment. The "forces" in the adjustment equations for L, K, X in (7), (8) can also be interpreted by Hooke's law because in every force there is a deviation from the equilibrium state.

5. Results of the Modelling

Our model consists of Eqs. (7-11) with 6 endogenous variables. Only the graphs of numerical solutions of the differential equations are presented because an exact solution for this non-linear system either does not exist or is very complicated. The solution of the model in Eqs. (7-11) with $A_L = 20, a = 800, B_L = 0.05, b = 0.1, c = 100, s = 0.1, k = 0.02, m_X = 10, m_L = 10, m_K = 20, v = 0.08, H = 0.01, A_K = 30, B_K = 0.1$ with initial values$L(0) = 20, X(0) = 200, K(0) = 10, p(0) = 100, w(0) = 5, r(0) = 5$ is shown in Figure 10, where the production and profit functions of NFS are also presented. All time paths show fluctuations and a tendency toward a steady state. Price, wage, and profit have a positive and production a negative time trend. Others fluctuate around a relatively constant value. In consumption, the vulnerable local behavior disappears by increasing the scale of the axis.

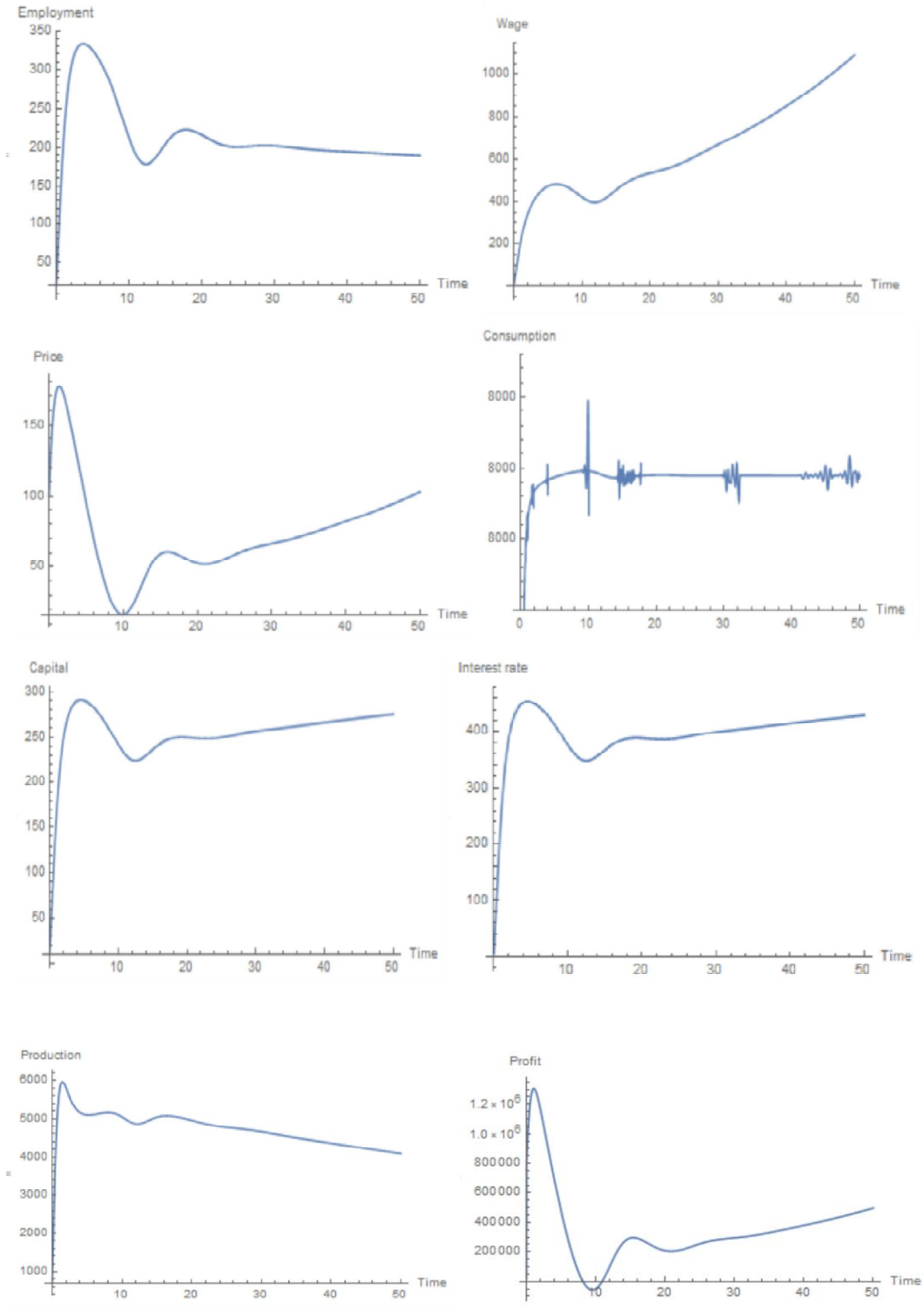


Figure 10. Time paths of $X(t), L(t), K(t), p(t), w(t), r(t)$, $\mathrm{Q}(t)$, and $\Pi_N(t)$.

To demonstrate the analyses that can be made with the model, we omit capital, consumer goods, and interest rate from the model. The model consists then of two equations (7), (10), where K, X, r are omitted and price is assumed fixed, $p = 40$. The profit function is then $\Pi_N(t) = p(A_L L(t) - (B_L/2)L^2(t)) - w(t)L(t)$. Assuming $A_L = 2, s = 0.01, m_L = 30$ in Eqs. (7) and (10) and first $B_L = 0.3$ and then $B_L = -0.015$ with initial values $L(0) = 2, w(0) = 5$, we get the solutions given in Figures 11-12. In the case of decreasing marginal productivity of labor, all time paths are stable (Fig. 11) while increasing marginal productivity of labor makes all time paths increasing without limit (Fig. 12). Thus, flexible wage does not stabilize this system.

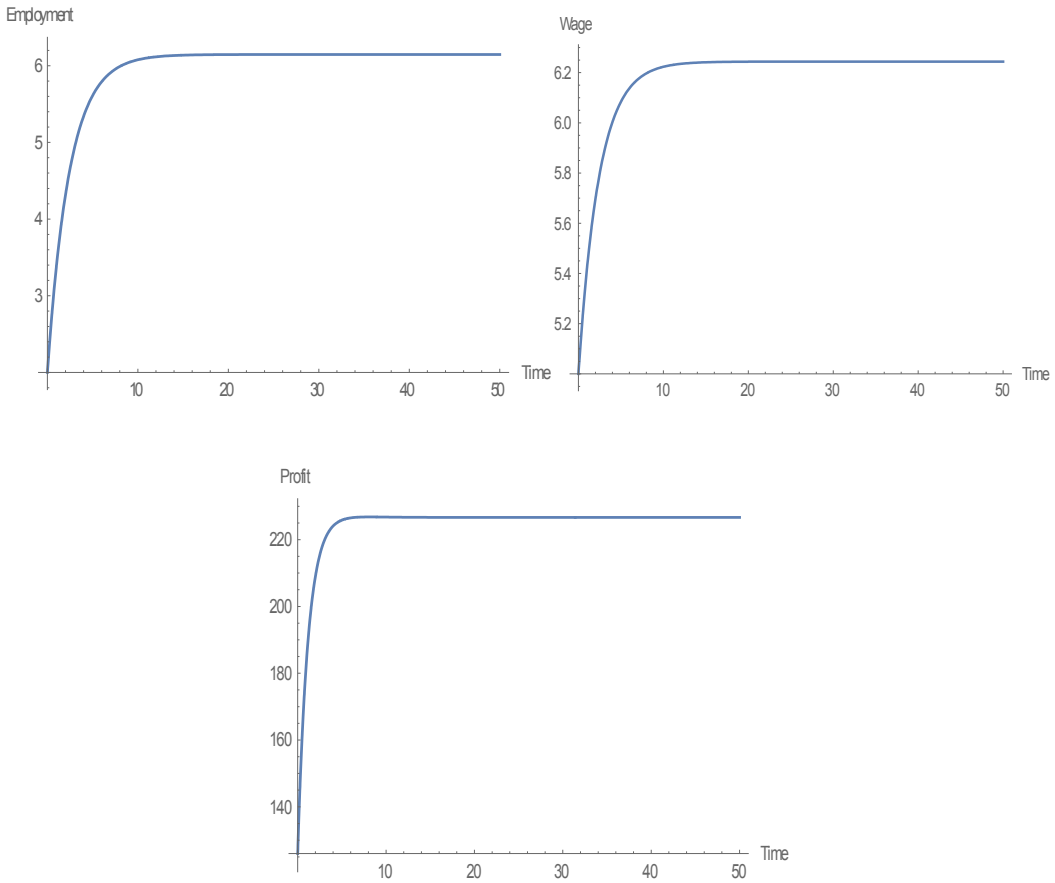


Figure 11. Employment, wage, and profitpath with decreasing marginal productivity of labor.

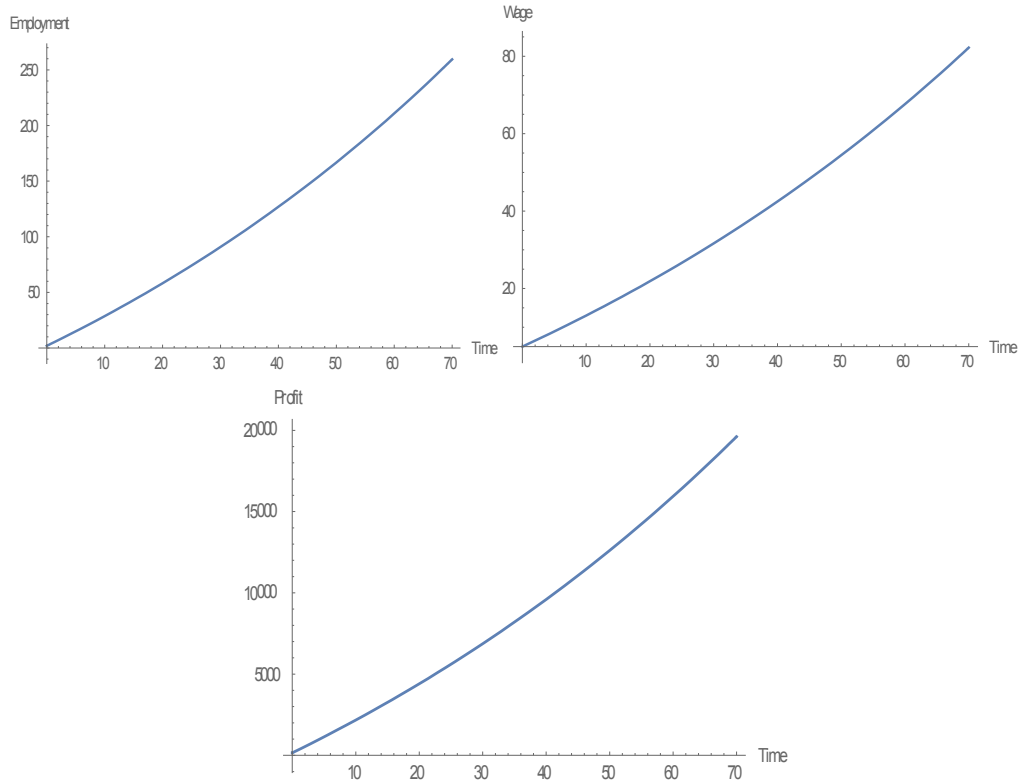
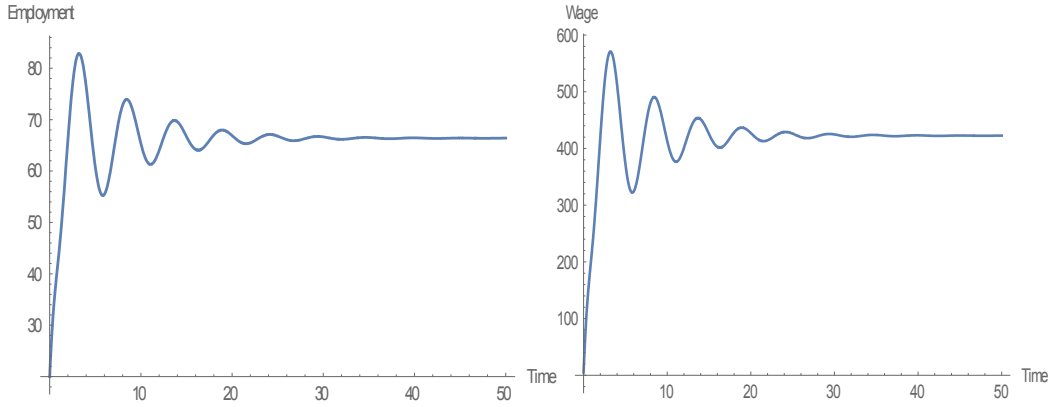


Figure 12. Employment, wage,
and profit path with increasing marginal productivity of labor.

However, adding Eqs. (8), (9) in the system with $A_L = 120, B_L = -0.015, a = 0.8, b = 0.0001, s = 0.3, m_L = 30, m_X = 30, k = 0.003,$ $c = 100$ with initial condition $L(0) = 20$, $X(0) = 20$, $p(0) = 10$, $w(0) = 5$, it is shown in Figure 13 that the system is stable even with increasing marginal productivity of labor. Thus, flexible price level is essential for the stability of the system.



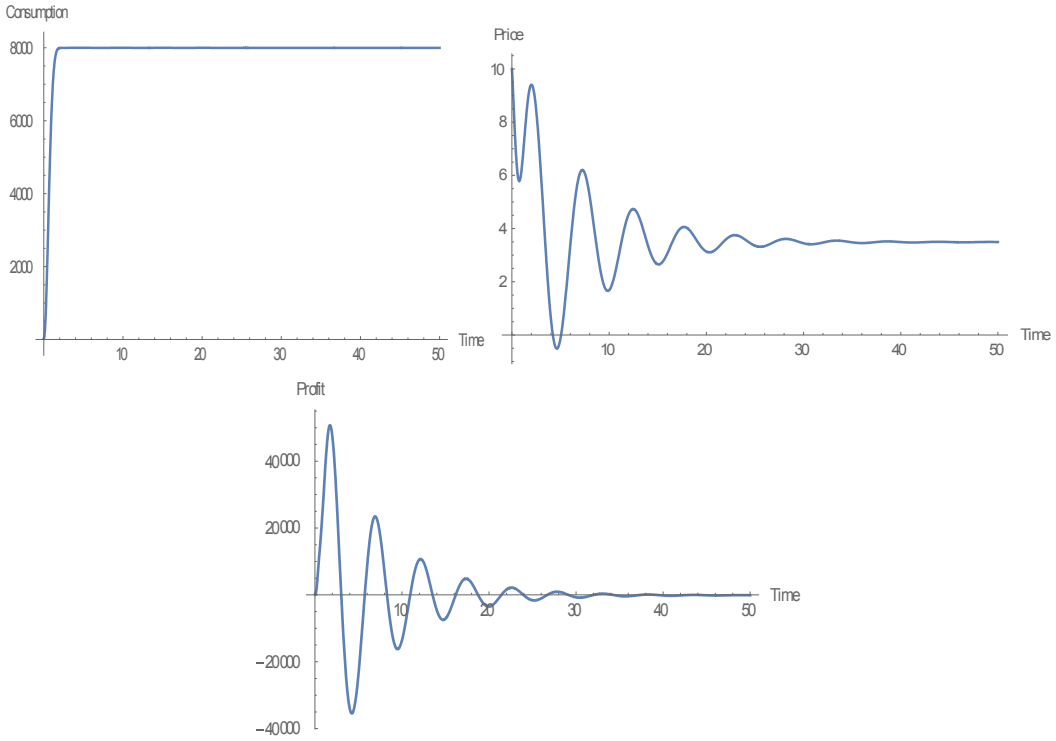


Figure 13. Time paths with increasing marginal productivity of labor.

6. Conclusions

We introduced a dynamic macro-model consistent with National Accounts. We concentrated on interactions between Non-Financial Firms' Sector (NFS) and Households' Sector (HS) and omitted the other sectors to keep the model tractable. NFS was assumed to behave in a profit-seeking and HS in a utility-seeking way. Production, employment, investment, and consumption decisions of the sectors were defined on this basis. Price was assumed to adjust according to the excess demand of consumption goods, wage was assumed to adjust according to the difference between the monetary value of marginal productivity of labor and wage, and interest rate was assumed to adjust according to the difference between the monetary value of marginal productivity of capital and interest rate. This gave a system of six differential equations with various kind of linkages between the variables and the sectors. Because an explicit solution for such non-linear dynamic system does not exist or is very complicated, we demonstrated the solutions by the time paths of the variables.

In general, oscillatory behavior was observed for all variables and the time paths turned out to be stable in most cases. However, different

parameter values gave also unstable behavior. By assuming only labor input in production with decreasing marginal productivity, the system was shown to be stable. Increasing marginal productivity of labor, however, made the system unstable with flexible wage level. In the case of flexible wage and price, stability was obtained. This shows that the flexibility of price and wage are essential for the stability of economic systems. Possible reasons for economic growth could have been studied by the model by assuming time dependencies in different parameters. These analyses were, however, left for the future.

Our study stresses the difference between micro- and macroeconomic modelling. In microeconomics, the only feedback effect taken into account in firms' production and pricing decisions is other firms' or consumers' reactions. In macroeconomics, firms' employment and production decisions affect the wage and the price levels in the economy that have feedback effects via consumers' decisions on firms' profitability. Thus, the feedback effects between firms and consumers are much more importantin macroeconomic modelling.

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