

APPLICATIONS ON MARKET RISK MANAGEMENT BY USING THE BAYESIAN APPROACH

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Abstract. *In the last decades, the Bayesian probability theory has become widespread in many areas of science and social sciences such as marketing. Bayes' theorem allows decisions and evaluating market research under uncertainty and limited data. The Bayesian approach is used in decision-making when there are high levels of uncertainty and limited information on which to base decisions, when there are available only expert opinions or data from previous time periods. It is easier to understand for business people who are less familiar with the understanding of mathematics, statistics or econophysics. Bayesian methods are actually some sort of bridges between classical methods of business and statistics in order to make the best decision. This article is a sum of applications on market risk management by using the Bayesian approach, for companies striving to enter foreign international markets.*

Keywords: *Bayesian theory, risk management, international marketing.*

1. Introduction

Internationalization is a progressive process that takes place gradually, translation from local to global market being slow. Expanding business in international markets is a sine qua non condition for every company that aims at developing, at benefiting from certain advantages that the domestic market lacks. Until the great energy crisis of the 70s, the international market entry strategies were based on the premise that the economic environment was in continual, linear ascending change, and highly predictable by extrapolating data from the previous period. The two oil shocks of the 70s and later, the financial global crisis from 2008-2009 changed the vision of companies on risk management science.

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Companies analyze the strategic options that they have for entering international markets and for this purpose they have a variety of investment options, either acquisition of assets or current expenses, for obtaining future benefits. The investment process involves several economic compromises, between exposure to some potentially unfavorable conditions, on the one hand and the anticipated return on investment on the other. Typically, the higher profitability, the greater the exposure to risk. In general, one can say that firms investing abroad apply a prospective, innovative and growth-oriented strategies, in search of new markets and opportunities, taking also some risks.

It should be noted, however, that determining the economic benefits resulted from investments is a complex process, making difficult to determine the exact additional cash flows arising from investments as the effects of investment may not be reflected separately in the financial statements of the company, which are usually, combined with other accounting information. In practice, it is impossible to anticipate all the investment opportunities because planning is done in an unstable and evolving environment, that, sometimes, between the budgeting and the implementation, conditions may change dramatically. The best investment decision should be taken in the context of wider strategy, risks, competitor reactions and managerial capacity to implement investment.

The analysis of investment alternatives requires consideration of several factors, such as: defining problems, the nature of the investment, cost and benefits of future growth effect of cash -flows, relevant accounting information and the costs incurred thereafter.

When a company decides to invest abroad, it had to take into account a variety of risks, classified according to several criteria: the impact, the probability, or likelihood of occurrence, the degree of diversification, etc. The issue of risk is very complex, requiring a multicriterial and multidisciplinary approach, which would require the expertise of different specialists, such as economists, statisticians, marketers, lawyers, computer scientists, policy analysts, scholars, etc.

The study of risk management started after World War II. Risk management has long been associated with the use of market insurance to protect individuals and companies from various losses associated with accidents. Supplementary methods of risk management assessment, emerged during the 1950s when market insurance was perceived as very pricey and imperfect for protection against pure risk. The use of derivatives arose during the 1970s, and expanded quickly during the 1980s. International risk regulation began in the 1980s, and banks or

companies settled internal risk management models and capital calculation formulas to hedge against unanticipated risks and reduce regulatory capital. After 2000, governance of risk management became indispensable, integrated risk management was introduced, and the chief risk officer positions were created. However, all these things failed to prevent the financial crisis that began in 2007.

The risk has two main components, for a given event:

- Probability of occurrence of the event;
- The impact of this occurrence (the magnitude of the risk).

For any event, the risk can be defined according to probability and impact. The risk has two main components, for a given event:

$$Risk = f(probability, impact).$$

On the other hand, the risk of an unfavorable event is based on a cause. Causes can be foreseen, unforeseen or accidental. Risk reduction can be done through identifying the source and the measures to counteract the unwanted event. Thus, the risk can be also defined by the relationship below:

$$Risk = f(hazard, measures).$$

The risk is determined by a multitude of objective causes, namely: change of economic conditions over time, rapid technological changes, invalidation of previous experience, imperfect knowledge of exogenous variables, the optimistic or pessimistic attitude of the analysis team, errors of technical or economic analysis, state interventions, the impact of the environment, price changes, modifications of foreign exchange rates, etc.

Two major categories of risks are defined in the literature:

“Pure” risks – which are the consequence of accidental or accidental events (hurricanes, fires, earthquakes, floods, wars, attacks, etc.)

“Speculative” risks - related to the decisions that are made in an enterprise, which make up a lot of events whose probability of occurrence is higher, the larger the enterprise is.

The total risk of an investment is the sum of several risks such as: commercial risk, financial risk, technical risk, established as a product of factors, corresponding to the operating probabilities of the sub-assemblies of the complex product or the technological system, social risk, related to the social consequences of some projects or programs, such as those related to unemployment, social conflicts, strikes, due to different mentalities, traditions.

The theorem of the English mathematician and Presbyterian minister Thomas Bayes (1702-1761) is essential for statisticians, as it provides a

mathematical framework to form conclusions by using probabilities. The evaluation of the various states is expressed in terms of degrees of belief estimates or related the evolution of a phenomenon numerically evaluated through subjective probabilities.

In the last decades, the Bayesian probability theory has become widespread in many areas of science and social sciences. Bayes' theorem allows taking decisions and market research under uncertainty and limited data.

The Bayesian approach is used in decision-making when there are high levels of uncertainty and limited information on which to base decisions when there are available only expert opinions or limited data from previous time periods. It is easier to understand for business people who are less familiar with the understanding of mathematics, statistics or econophysics. Bayesian methods are actually bridging classical methods of business and statistics in order to make the best decisions.

Decision analysis consists of two stages. In the first stage, researchers specify the logical structure of the decision problem in terms of producing random events and sequential decisions, using for this purpose the decision tree, which is a diagram that connects the initial decision (tree trunk) with the final results (branches of the tree).

The second stage consists of the reverse process, starting from branches to the trunk, in order to calculate the expected values of the random events and removing branches (variants) with low net anticipated benefits.

When there is a partial lack of information about the target market, i.e. when the marketer has a set of information about the market, one can use the Bayesian decision trees method (from Th. Bayes). It combines statistical test results with other data, thus creating an integrated background for decision making.

In probability theory and statistics, Bayes' theorem relates current probability to prior probability and is used in the mathematical manipulation of conditional probabilities.

The theorem expresses how a subjective degree of belief should reasonably change to account for evidence: this is Bayesian inference, and has applications in a wide range of domains.

Bayes' theorem is stated mathematically as the following equation:

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)},$$

where A and B are events.

- $P(A)$ and $P(B)$ are the probabilities of A and B without regard to one other.
- $P(A | B)$, a conditional probability, is the probability of A given that B is true.
- $P(B | A)$, is the probability of B given that A is true.

Bayes' theorem has many forms, and I shall use some of them in this article. Several case studies analyzing decision options related to launching of new products on target markets will be presented in the following.

2. Applications on market risk management by using the Bayesian approach

Thus, in the following, let's consider the case of a company that analyzes the option of entering a new foreign market, with a brand-new product. It has to choose between two alternatives:

- to start production and find a method of entering a market;
- to give up producing and launching on an overseas market.

Applying the decision theory, four parameters will be used:

Actions:

A1: introduction product manufacturing and marketing, the target string;

A2: waiver of placing the product into production and market launch – target string;

States of the nature:

S1: favorable market reaction;

S2: unfavorable market reaction.

a) The a priori model

The managers think that if the new product will be launched, and the purchasers will have a favorable reaction, a profit of \$ 550,000 will result, and if the market reaction will be unfavorable, the loss will be \$ 225,000.

The results are estimated by specialists with experience in marketing. In this model, the chances, in either of the variants, will be assessed subjectively. In this case study, managers consider the assumption of a favorable market reaction of 0.3, and the probability of an unfavorable market reaction of 0.7.

Table 1.
Calculation of the mathematical expectancy of financial results (MFE).

Alternative	Probability of the favorable reaction of the market $P(S_1)$	Consequences of the favorable reaction of the market	Probability of the unfavorable reaction of the market $P(S_2)$	Consequences of the unfavorable reaction of the market	Mathematical financial expectancy (MFE)
Action A1: Manufacturing a new product and launching it on a foreign market	0.3	550.000\$	0.7	-225.000 \$	7.500 \$
Action A2: Giving up manufacturing a new product and launching it on a foreign market	0.3	0 \$	0.7	0 \$	0 \$

The mathematical financial expectancy (MFE) if managers decide to manufacture a new product and launching it on a foreign market (action A1) is $\$ 7.500 = (0.3) (\$550,000) + (0.7) (\$ -225.000)$.

If managers renounce to manufacture of the new product (A2), the mathematical expectation becomes:
 $0 \$ = (0.3) (0\$) + (0.7) (0\$)$.

The decision tree, presented in Figure 1, shows clearly that the selected action will be A1 because its mathematical expectation is higher than that of A2.

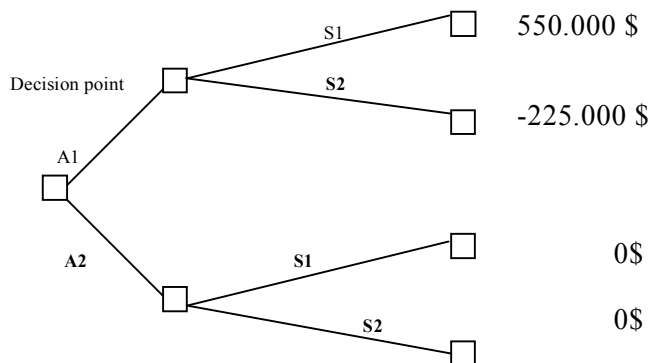


Figure 1. Decision tree related to manufacturing a new product and launching it on a target-market.

b) The developed model, after obtaining additional pieces of information

The apriori model, shown above, is based on a great uncertainty. Therefore, a decision should be taken after the evaluation of the target market reactions.

In the following, we assume that one can decide concluding a contract worth \$ 15,000 for a market research with a 75 % statistical certitude, regarding the foreign market reaction towards a new product. The result of the survey confirms or not the demand for the product, that will be positive (R1) or negative (R2).

The probability of firm demand for the new product, even if there search will reveal a favorable reaction of the market of 0.75 ($R_1 | S_2$) and only 0.25 ($R_2 | S_1$) if the research will not reveal a firm demand for the product, even if there market reaction would be favorable.

The compound probability to achieve simultaneously the result R1 of the survey (firm request for the new product) and the states of nature S_1 and S_2 , is the following:

$$P [R_1 \cap S_1] = P (S_1) \times P (R_1 | S_1) = (0.3) \times (0.75) = 0.225$$

$$P [R_1 \cap S_2] = P (S_2) \times P (R_1 | S_2) = (0.7) \times (0.25) = 0.175.$$

Conversely, if the market research will identify the result R2 of the survey (lack of strong demand for the new product), the compound probability to achieve simultaneously the result R2 and the states of the nature S_1 , respectively S_2 is the following:

$$P [R_2 \cap S_1] = P (S_1) \times P (R_2 | S_1) = (0.3) \times (0.25) = 0.075$$

$$P [R_2 \cap S_2] = P (S_2) \times P (R_2 | S_2) = (0.7) \times (0.75) = 0.525.$$

By adding the compound probabilities one can obtain the marginal probabilities:

$$P (R_i) = [R_i \cap S_1] + [R_i \cap S_2]$$

$$P (S_j) = [S_j \cap R_1] + [S_j \cap R_2].$$

Finally, the posterior probabilities can be calculated for S_1 and S_2 . Posterior probabilities are conditional probabilities on achieving S_j event in the conditions of occurrence of another event, R_i , which is actually the result achieved after obtaining new pieces of information. Applying Bayes ' theorem, the posterior probabilities for R_1 and R_2 are calculated as follows:

$$P(S_1 | R_1) = 0.5625$$

$$P(S_2 | R_1) = 0.4375$$

$$P(S_1 | R_2) = 0.1250$$

$$P(S_2 | R_2) = 0.8750$$

$$P(S_1 | R_1) = \frac{P(S_1) * P(R_1 | S_1)}{\sum_{j=1}^n P(S_j) * P(R_1 | S_j)} = 0.5625$$

$$P(S_2 | R_1) = \frac{P(S_2) * P(R_1 | S_2)}{\sum_{j=1}^n P(S_j) * P(R_1 | S_j)} = 0.4375$$

$$P(S_1 | R_2) = \frac{P(S_1) * P(R_2 | S_1)}{\sum_{j=1}^n P(S_j) * P(R_2 | S_j)} = 0.1250$$

$$P(S_2 | R_2) = \frac{P(S_2) * P(R_2 | S_2)}{\sum_{j=1}^n P(S_j) * P(R_2 | S_j)} = 0.8750$$

Table 2.

*The compound, posterior and marginal probabilities
in the case of entering a new foreign market with an absolutely new product*

	Compound probabilities with		Marginal probabilities P (R _i)	Posterior probabilities for	
	S ₁	S ₂	(col.1 + col.2)	S ₁ (col.1: col.3)	S ₂ (col.2: col.3)
0	1	2	3	4	5
The results R ₁ of the research	0.225	0.175	0.40	0.5625	0.4375
The results R ₂ of the research	0.075	0.525	0.60	0.1250	0.8750
The marginal probabilities P (S _j)	0.30	0.70	1.00		

Introducing the posterior probabilities model is very utile in decision making because they help computing the mathematical expectation for decision alternatives according to the results of the market research. If the study reveals a firm demand (result R1) for decision making process to enter new foreign market with an absolutely new product, we can calculate the mathematical expectation of the financial results of manufacturing and marketing a new product on a foreign target market, as follows in Table 3:

Table 3.

Calculation of the mathematical expectancy of each action, when market research reveals a firm demand (result R1) for decision making process to enter an unknown foreign market with an absolutely new product

Action alternatives	P (S ₁ R ₁)	The value of favorable reaction of the market (state S ₁)	P (S ₂ R ₁)	The value of favorable reaction of the market (state S ₂)	MFE (col.1 x col.2) + (col.3 x col.4)
0	1	2	3	4	5
A₁ : Manufacturing the new product and launching it on the target market	0.5625	550.000 \$	0.4375	-225.000 \$	210.938 \$ = (0.5625 x 550.000 \$) + [(0.4375) x (-225.000 \$)] = 309.375 \$ - 98.437 \$
A₂ : Renouncing to the manufacturing of the new product and launching it on the target market	0.5625	0 \$	0.4375	0 \$	0 \$

In other words, if the selected action will be A1 (product introduction in manufacturing and launching it on the target market) as mathematical expectation of financial results of A1 is bigger than A2.

However, if we study shows the absence of firm demand (result R2) for decision making process to enter on anew, unknown foreign market, with an absolutely new product, we can calculate the mathematical

expectation of the financial results for manufacturing a new product manufacturing and launching it on a new foreign market, as follows in Table 4:

Table 4.

Calculation of the mathematical expectation of each action when it shows the absence of firm demand (R_2 result) in the decision-making process of entering an unknown external market with an absolutely new product

Action alternatives	$P(S_1 R_2)$	Value of favorable reaction of the market (state S_1)	$P(S_2 R_2)$	Value of favorable reaction of the market (state S_2)	MFE (col.1 x col.2) + (col.3 x col.4)
0	1	2	3	4	5
A₁: Manufacturing the new product and launching it on the target market	0.1250	550.000 \$	0.8750	-225.000 \$	-128.125 \$ = (0.1250 x 550.000 \$) + [(0.8750) x (-225.000 \$)] = 68.750 \$ - 196.875 \$
A₂: Renouncing to the manufacturing of the new product and launching it on the target market	0.1250	0 \$	0.8750	0 \$	0 \$

This time, the selected action will be A_2 (waiving manufacturing the new product and launching on a foreign target market) as mathematical expectation of the financial results of A_2 is greater than that of A_1 . It is preferable to have zero profit, then to get a loss of \$ -128.125.

Lastly , the calculation of mathematical expectancy of the financial results obtained for the value obtained in the case of the market research will be calculated according to the formula:

$$P(R_1) \times \text{MFE}/R_1 + P(R_2) \times \text{MFE}/R_2 =$$

$$(0.40) \times (210.938 \$) + (0.60) \times (0 \$) = 84.375 \$.$$

This is the *probable value of the additional information*, resulted after the market research. Based on this result, one can calculate the difference between the mathematical expectation of the financial results in the case of not contracting a market research (\$ 7.500) and the mathematical expectation of the financial results of the value obtained in the case of contracting a market research (\$ 84.375), representing virtually the value of the new pieces of information obtained after the research: \$ 76.875.

By deducting from this result the cost of the research, of \$ 15,000, we will obtain *the probable net value of the new information* obtained from the market research, of \$ 61,875. It reflects the investment effort that the company must make in order to obtain new information.

Under these conditions, the conditional probabilities $P(R_1 | S_1)$ and $P(R_2 | S_2)$ would become 1, instead of 0.75, and $P(R_1 | S_2)$ and $P(R_2 | S_1)$ would become 0 instead of 0.25.

Sometimes marketers also use the notion of "*perfect information*". It starts from the ideal hypothesis that the information that will be obtained from the research will have a 100% certainty percentage.

Under these conditions, the conditional probabilities $P(R_1 | S_1)$ and $P(R_2 | S_2)$ would become 1, instead of 0.75, and $P(R_1 | S_2)$ and $P(R_2 | S_1)$ would become 0 instead of 0.25.

Therefore, the compound probabilities for R_1 and R_2 would also change, they would transform from 0.225, respectively 0.175 to 0.3 and 0, according to the formulas:

$$P[R_1 \cap S_1] = P(S_1) \times P(R_1 | S_1) = (0.3) \times (1) = 0.3$$

$$P[R_1 \cap S_2] = P(S_2) \times P(R_1 | S_2) = (0.7) \times (0) = 0.$$

Of course, the marginal probabilities attributed to R_1 and R_2 will be equal to the a priori probabilities of 0.3 and 0.7 of S_1 , respectively S_2 .

Thus, the mathematical expectation of the financial results under conditions of certainty will become:

$$\text{– for the action } A_1: (0.3) (550.000 \$) + (0.7) (0 \$) = 165.000 \$.$$

$$\text{– for the action } A_2: (0.3) (0 \$) + (0.7) (0 \$) = 0 \$.$$

So, under ideal conditions, of 100% certain information, if the R_1 result is obtained, automatically S_1 is obtained, as a state of nature and we will choose the A_1 action with the MFE of \$ 165,000, and if the R_2 result is obtained, automatically S_2 will be obtained, as a state of nature and we will choose action A_2 with MFE of \$ 0.

The probable value of the perfect information will be \$ 165,000- \$ 7,500 = \$ 157,500, in fact the cost of uncertainty regarding the selection

of action A_1 , when we do not have the perfect information (we have not invested in market research). If we decide to spend funds for the survey, the value of the research contract must be at most \$ 157,500.

In the chosen case study, when the research costs only \$ 15,000, it is obvious that it is more than advisable not to avoid investing in a professional survey, which would be of real benefit to the company that wants to enter the foreign market.

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