A PHASE TRANSACTIONS OF THE USD/JPY EXCHANGE RATE WITH MONETARY POLICY

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Abstract. *In this paper, the USD/JPY exchange rate is modeled by anon-linear differential equation (NDE), which consists of the trend noise only which is called a trend noise. The most significant finding is that, based on this model, conflicts of currency policies between the US and Japan occurs whenever large errors between the model and a real exchange rate (RER) emerge. The latest error is caused by the BoJ's ultra-loose monetary policy, which leads to strong depreciation of the yen. Additionally, I discuss why the RER is staying difficult on the around an equilibrium exchange rate (EER), caused by the even simple noise which is called a white noise.*

Keywords: *Nonlinear Differential Equation (NDE), Noise Analysis, Real Exchange Rate (RER), Equilibrium Exchange Rate (EER), Monetary Policy.*

JEL classification: C30, C61, E52, F31

Even Adam Smith, the Canny Scot whose monumental book, "The Wealth of Nations' (1776), represents the beginning of modern economics or political economy - even he was so thrilled by the recognition of an order in the economic system that he proclaimed the mystical principle of the "invisible hand": that each individual in pursuing his own selfish good was led, as if by an invisible hand, to achieve the best good of all, so that any interference with free competition by government was almost certain to be injurious. --Paul Samuelson (1948).

1. Introduction

An equilibrium exchange rate (EER) is estimated by many models. "A foreign exchange rate is the price of one currency in terms of another, which is typically set in financial markets by supply and demand for currencies." McCown, Pollard and Weeks (2007) says. "Supply and demand for currencies are strongly influenced by conditions in the market for other financial assets, as

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well as market perceptions of underlying real macroeconomics conditions. To be described the EER with long-term, there are of course a lot of approaches with econometric models based on fundamentals." The approaches can be classified into three main types: 1) Purchasing Power Parity (PPP), 2) Behavioral Equilibrium Exchange Rate (BEER), 3) Fundamental Equilibrium Exchange Rate (FEER). PPP was first introduced by Cassel (1916 and 1918). The definition of the exchange rate is a ratio of home and foreign prices. The BEER approach, modeled by Clark and McDonald (1998), provides a real exchange rate (RER) and an explanation for the slow reversion to the PPP, and the exchange rate is determined as an actual value of economic fundamentals. The FEER, formalized by Williamson (1983 and 1994), is defined as adjusting the country's internal and external balance. It ignores the short-run cycle and concentrates on components that persist in the medium-term. Clearly, Clark and McDonald (1998) explain the difference between the BEER and the FEER; the BEER aims to assess the EER by using a more empirical approach based on fundamental variables affecting the exchange rate on the short and medium term – it denotes a modeling strategy which attempts to explain the actual behavior of the exchange rate in terms of relevant economic variables. In contrast, the FEER is designed to calculate the medium-term real effective value to assess the current value of the exchange rate. Obstfeld and Rogoff (2005) and Blanchard, Giavazzi, and Sa (2005) state the notion of the EER with current account changes figured prominently in global imbalance.

To predict future exchange rate (FER), the two main methods of modeling the exchange rate are via macroeconomic fundamentals (MF) and random walk (RW). The MF is described by money supplies, outputs, inflation, interest rates, etc. Meese and Rogoff (1983a, b) use the MF and traditional economic theory to model the FER. Yoshimori[1] (2003) suggests that the EER and the FER can be described by MF. Nelson (1995) and Chinn and Meese (1995) conclude that the FER can be described by RW, on the other hand, seeing that the FER has misaligned widely against the price predicted by MF. Bellgard (1999) also discusses the FER based on RW hypothesis. Nevertheless, Kilian (2001) concludes that RW is more likely to remain elusive for the foreseeable future, and Rossi (2006) rejects the hypothesis of RW on the exchange rates. Additionally, Ca' Zorzi (2013) shows that a half-life Purchasing Power Parity (PPP) model can forecast the FER, rather than the RW model, both short-term and long-term.

[1] The model is a state-of-art econometric model with a dynamical system based on integrating fractal theory with traditional econometric methods; 1) by a measurement of a fractal dimension – how many factors should be described the USD/JPY exchange rate and 2) by two principle components – the exchange rate is described.

Notwithstanding, there are still not any models which have completely succeeded in modeling both the EER and the FER. A basic idea is that I try to integrate MF with RW by a non-linear differential equation (NDE) model. As an empirical result, the RER is reflected basically by MF (see the NDE in the next paragraph).

This paper uses the NDE model based on a dynamic system with a factor analysis to model the RER. For each time interval, I derive the differential equation from the time series of the exchange rate, whose solution approximates the data, including differentials up to 2nd-and 3rd-order. The NDE consists of combinations of the 2nd- and 3rd-order differential, through a low-pass filter, including a trend noise only and removing simple noise (white noise, i.e. pure randomness). In fact, the time series is influenced by the noise which is not white noise but trend noise (i.e. pink noise or Brownian noise) – a consecutive price actions such as appreciation or depreciation.

From the empirical analysis, an ultra-loose monetary policy by Bank of Japan (BoJ): Quantitative and Qualitative Monetary Easing (QQE), derived from Abenomics, compared with the monetary policy in the late 1990s to early 2000s[2], induces large errors, $\frac{d^3}{dt^3}\left(f(t)\right)$ in a stage of an "input" and the real exchange rate. The implication of large errors is that the monetary policy after Feb. 2014[3]including QQE2 leads to an accelerating depreciation of the yen. This may be a strategy of the Japanese government, namely, the Japanese economy is boosted by recovering profits from the export sector. This may suggest currency manipulation including a "beggar-thy-neighbor" policy.

The discussion is why the real exchange rate (RER) ignores the equilibrium exchange rate (EER), which is estimated in the medium-and long-term by economic theories. More faculty, the RER cannot hover around the EER walks unsteady and trajects speedy around the EER – for instance, the RER has misalignment against the EER – the Purchasing Power Parity (PPP). What causes the RER misalignment? The EER is unable to update quickly against updated information. Even so, why can't the market restore the EER quickly? Specifically, why do investors often forget or ignore the value of "fundamentals?" The answer is that the RER is weakly coiled around the EER so that the entropy of the RER around the EER is enlarged by white noise, which is produced by the errors between the NDE model and the RER. More concretely, walking the RER like RW is caused by white noise – psychology from technical chart analysis, a statement of central banks or executives, news headlines, a geopolitical risk or an option price – and leads to a shifted price range.

[2] See Ito (2005)

[3] https://www.boj.or.jp/en/announcements/release_2014/k140218a.pdf

Accordingly, it would be difficult for the RER to hover an equilibrium point such as the EER.

The remainder of the paper is organized as follows. Section 2 describes the data and calibration. Section 3 explains the methodology (3-1) and empirical results: monetary policy and exchange rate (3-2)**.** In Section 4, I discuss: the RER cannot be hovering on the EER. The last section contains concluding remarks.

2. Data and Calibration

My empirical analysis is based on the monthly USD/JPY exchange rate time series, obtained from Bloomberg (see Fig. 1). The data x_t were sampled monthly (from October 1985 to December 2017). I selected the end date to coincide with the Plaza Accord on September 22, 1985, which embodied a new regime, although the actual shifting from fixed to floating exchange rates occurred with the Smithsonian Agreement in 1973 after the Bretton Woods Agreements in 1971.

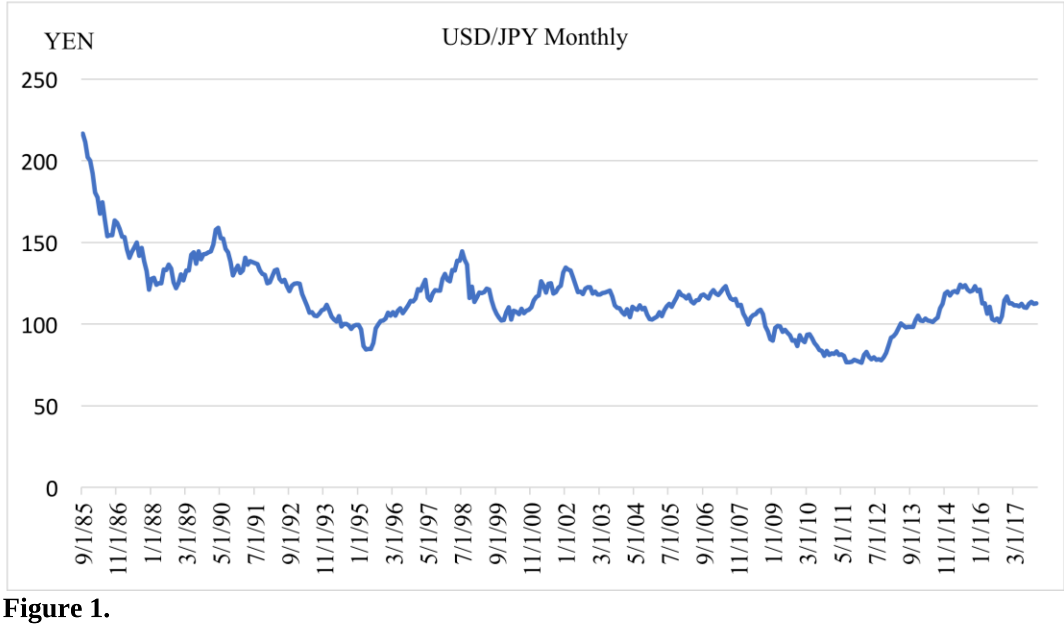

Figure 1.

3. Methodology and Empirical Results

3.1. Methodology

In a noise analysis, two popular noises, i.e., both white noise with the power spectrum $s(f) \sim f^0$, andpsychological noise (also known as 1/f noise) with the power spectrum $s(f) \sim f^2$ are very well know and understood. Additionally, the noise is ubiquitous in financial markets as well as in physics.

More factually, the time series of the monthly USD/JPY exchange rate is not simply by a white noise (i.e. a pure randomness), but a trend noise (i.e. pink noise or Brownian noise). Vandewalle and Ausloos (1997) shows that the foreign exchange currency rates do not obey the Brownian motion (white noise) rule, but is rather a fractional Brownian motion (see West and Deering (1995)). The time series is formed by the trends, which are consecutive of appreciation or depreciation caused by trend noise, so trend noise has a better fitting than white noise (see Yoshimori 2001).

Assume that the time series S_t of the monthly USD/JPYis composed additively from a long-term signal s_t including trend noise and white noise ε_t, that is $S_t = s_t + \varepsilon_t$. Alow-pass filter is expected to produce S_t, an estimation of the long-term signal s_t by removing ε_t. The implication is that the long-term signal is formed by political accord, monetary policy, trade policy, and macro fundamentals, while white noise is influenced various daily sources such as rapidly changing political risks, unexpected informal shocks to investment opportunities, etc.. To remove white noise, Oppenheim and Schafer (1989) have suggested a simple low-pass filter, and Guo, Wang and Bell (2002) have proposed the low-pass filter with Fourier transform (see Appendix A), and the white noise reduction is based on it.

Kaulakys and Alaburda (2009) present between relationship between 2nd-order structural NDE and pink noise or Brownian noise with the power-law distributions, however. Through the factor analysis (see Appendix B), the errors between a non-linear differential equation (NDE) model and a real exchange rate (RER) can be decreased remarkably by combinations of the 2nd- and 3rd-order differential rather than only 2nd-order one[4], because the 3rd-order one contributes to a bigger factor of the time series. In terms of a real word, the NDE including combinations of the 2nd- and 3rd-order differential could reflect more the long-term signal which is formed by economic policy and macro fundamentals. Additionally, the NDE includes 2nd-order structural differential

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[4] The 2nd-order structural NDE is $\frac{d}{dt}\big(f(t)\big) = (100 - x) \cdot \frac{d^2}{dt^2}\big(f(t)\big)$, and the peak of errors is as same as the 2nd- and 3rd-order structural NDE.

equation. The model is based on the standard theory of differential equations, through the factor analysis (see Appendix B). By combinations of the 2nd- and 3rd-order differential, the errors between a non-linear differential equation (NDE) model and a real exchange rate (RER) can be decreased remarkably, and I use the following method to construct the NDE.

Let $f(t)$ denotesthe monthly USD/JPY exchange rates. Given the exchange rate $f(t)$, I take the partial derivatives $\frac{d}{dt}\left(f(t)\right)$, $\frac{d^2}{dt^2}\left(f(t)\right)$, $\frac{d^3}{dt^3}\left(f(t)\right)$, etc. as additional data sets to perform the factor analysis of the exchange rate.

For given data sets $\frac{d^2}{dt^2}\left(f(t)\right)$, $\frac{d^3}{dt^3}\left(f(t)\right)$, and $f(t)$, the core part of the factor analysis is to express $f(t)$ as a nonlinear combination of $\frac{d^2}{dt^2}\left(f(t)\right)$, $\frac{d^3}{dt^3}\left(f(t)\right)$ that is, to write $f(t)$ as

$$f(t) = a\left(\frac{d^3}{dt^3}\left(f(t)\right)\right)^2 + b\,\frac{d^2}{dt^2}\left(f(t)\right)\cdot\frac{d^3}{dt^3}\left(f(t)\right) \qquad 3.1$$

allowing for error terms. In practice, the coefficients a and b are chosen to minimize the L^2 norm (see Appendix C) of error terms, i.e. the difference $\left[f(t) - \left\{a\left[\frac{d^3}{dt^3}\left(f(t)\right)\right]^2 + b\,\frac{d^2}{dt^2}\left(f(t)\right)\cdot\frac{d^3}{dt^3}\left(f(t)\right)\right\}\right]$ over the time interval under consideration.

In finding a and b, I consider up to the third derivative $\frac{d^3}{dt^3}\left(f(t)\right)$. The shifted data are then smoothed by suitable moving averages of $\frac{d^3}{dt^3}\left(f(t)\right)$. The moving average of $\frac{d^3}{dt^3}\left(f(t)\right)$ is a data set $MA(t)$ obtained from $\int_t^{t+a}\frac{d^3}{dt^3}\left(f(t)\right)$, which reduces the power $\sqrt{a^2(n)+b^2(n)}$ of the trigonometric function $a(n)\sin(nt)+b(n)\cos(nt)$ to $1/n$, that is, the power of the moving averaged trigonometric function $a(n)\sin(nt)+b(n)\cos(nt)$ is less than $\sqrt{a^2(n)+b^2(n)}/n = original\ power/n$. This process is the low-pass filter with the Fourier transform.

Since the validity of the equation depends on the error, additionally, I expect that the error function describes the structure of reality by a suitable interpretation of the terms in the equation[5]. The general solution of NDE is shown the Appendix D.

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[5] My insight is that psychological phenomena can be described by differential equations, and my definitions are below. Investors' psychology (i.e. minds) are captured

3.2. Empirical Result: Monetary Policy and Exchange Rate

From the data, I deduced the following differential equation for the smoothed monthly time series of the USD/YEN exchange rate:

$$f(t) = \left(\frac{d^3}{dt^3}(f(t))\right)^2 + 2\frac{d^2}{dt^2}(f(t)) \cdot \frac{d^3}{dt^3}(f(t)) \qquad 3.2$$

To justify that the above is optimized, I compare the L^2 norm of $\frac{d^2}{dt^2}(f(t)), \frac{d^3}{dt^3}(f(t))$; the former is 0.03 while the latter is 0.10.

Figure 2 shows the smoothed square is controlled within the indicated range of the Equation 3.2.

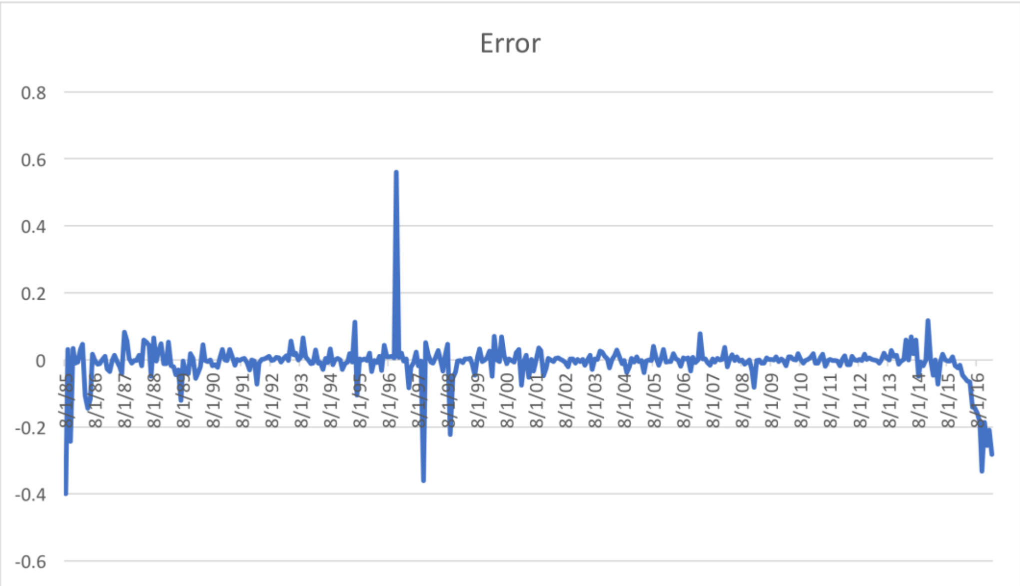

Figure 2.

The NDE includes the term of $\frac{d^3}{dt^3}(f(t))$. In physics, this term is called “jerk”. In economics, this term captures a control system, such as a currency

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by the 1st-order differential. The up-and-down trend of market atmosphere created by investors’ psychology is captured by the 2nd-order differential. The control power of the market atmosphere is captured by the 3rd-order differential. A more detailed definition of market atmosphere is the following: Investors think that today’s price action is influenced by yesterday’s price action. If the US dollar appreciates today, for example, investors anticipate that tomorrow’s price will also appreciate.

policy by the US or Japanese government. More concretely, the control power is set by intervention, monetary policy, and political accord[6]. Figure 3 shows errors between $\frac{d^3}{dt^3}\left(f(t)\right)$ and the real exchange rate (RER). Large errors (above 0.6) are observed in 1985, 1987, 1988, 1996, 1997 and after 2013. The NDE includes the term of $\frac{d^3}{dt^3}\left(f(t)\right)$.

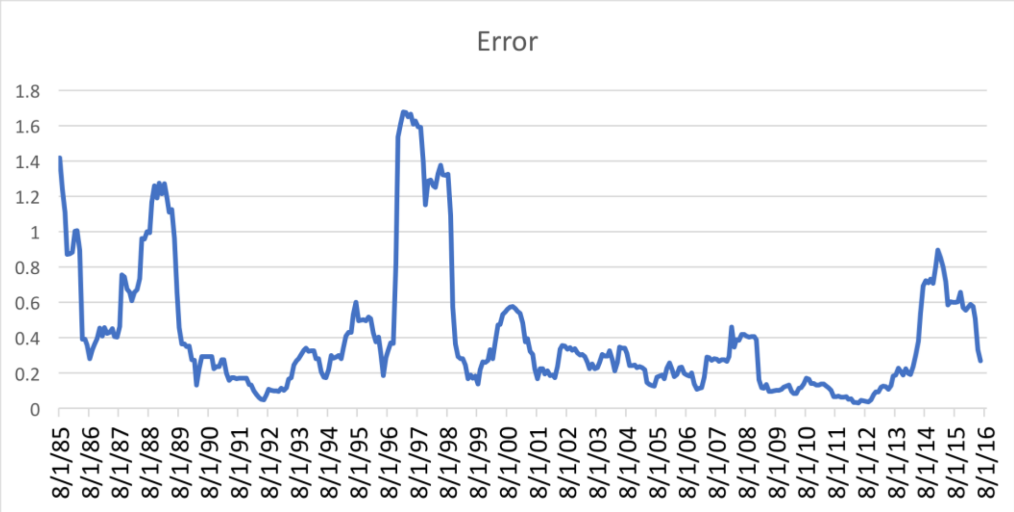

Figure 3.

I especially discuss the error after 2013. It seems that, indeed, the error is linked to the ultra-loose monetary policy by BoJ, to induce a depreciation of the Japanese yen – there is a relationship between the BoJ monetary policy and USD/JPY. In terms of both the differential interest rate and purchasing asset, the relationship between loosening or tightening monetary policy and depreciating or appreciating currency has been discussed elsewhere; the most familiar paper is Dornbush (1976)[7]. Eichenbaum and Evans (1995) indicates a strong empirical relationship between the federal funds rate (FFR) and the US

[6] James A. Baker III (2015), the former secretary of the US treasury, says that the middle term and long term exchange rate has been decided by political accords with fundamentals. Broz (2006) also says "Of course, governments cannot directly set the real exchange rate, but they can affect trends in the real exchange rate over a period long enough to be of political and economic significance – typically estimated at three to five years."

[7] Dornbush concludes that a monetary expansion brings temporarily to a large nominal depreciation.

dollar[8]. Brainerd (2016)[9] discusses the relationship between the FFR and the dollar index. In addition to these papers, unconventional monetary policy, like purchasing assets, has also affected the value of the dollar. For instance, Glick and Leduc (2015) find that the monetary policy has roughly three times the bang per policy surprise on the value of the dollar.

The Japanese yen, after trading in the 75-yen to 80-yen range to the US dollar in the autumn of 2012, has weakened to the middle 90-yen level amid market expectations for "aggressive monetary easing" under Prime Minister Shinzo Abe's administration. Bernanke (2017) says, "Abe's election the bank has effected a substantial easing in financial conditions, as reflected in the stock market, long-term interest rates, and the exchange rate[10]." Bénassy-Quéré (2014) also state that the experience of Japan, whose currency has greatly depreciated since late 2012 following the announcement of a massive expansionary monetary policy, illustrates the link between monetary policy and exchange rates[11].

From the empirical analysis, the errors term $\frac{d^3}{dt^3}\left(f(t)\right)$ enlarged under Abe's eco-political mind and the Quantitative and QQE1[12] on April 4, 2013.In addition, the error increased due to the predicted QQE2[13] on October 31, 2014.

The QQE1 could be evaluated reasonably – the government explains that the purpose is to escape from deflation of disinflation by depreciating the yen, even if the purpose of the policy is to boost competitiveness of Japan's exports. By the QQE2, however, 2% inflation was not produced, but only depreciation of the yen was induced. As evidence, the error of the $\frac{d^3}{dt^3}\left(f(t)\right)$ in a stage of

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8 The FOMC on February, 1994 announced a tightening monetary policy from increasing in FF rate from 3% on February 4, 1994 to 4.25% on May 17, 1994 and the dollar depreciated by 10.2% against the yen during February 4 to June 30.

9 "In particular, estimates from the FRB/US model suggest that the nearly 20 % appreciation of the dollar from June 2014 to January 2016 could be having an effect on U.S. economic activity roughly equivalent to a 200-basis point increase in the federal funds rate," Brainerd (2016) said. "Interestingly, it appears that this effect showed through in decreased business investment activity and stagnant manufacturing output, while the anticipated effect on net exports may have been somewhat dampened by depressed demand for imports of capital goods, among other factors."

10 The trade-weighted exchange rate has fallen from 107 to 86 (2010 = 100).

11 Neely (2010) says the policies of the Fed also appear to have resulted in a significant but moderate depreciation in the dollar.

12 https://www.boj.or.jp/en/announcements/release_2013/k130404a.pdf

13 BoJ explains about more committing expansion monetary policy, due to the sticky low inflation.

“input[14]” expanded after February 2014. In my view, the monetary policy doesn’t have a moderate tool as a domestic macro-policy for 2% inflation target – Forbs (2018) shows Japanese trend core CPI was not changed. Rather than the purpose of 2% inflation, it would probably cast doubt on currency manipulation including a “beggar-thy-neighbor” policy[15]. Accordingly, the BoJ should contemplate the 2013 G7 communique[16]; monetary policy should target the domestic objectives like inflation and the output gap, with taking a currency’s depreciation.

Of course, there is a similar discussion about the depreciated US dollar due to the Fed’s monetary policy. Indeed, the nearly 40% yen appreciation against the dollar between 2007 and 2012 caused by Fed monetary policy: QE1, QE2, and QE3, however. This does not correspond with currency manipulation, because the result is that the error is below 0.6 (see Fig. 3) and because the price action is along the Finished Goods Price Index (FGPI) (see Fig. 4 and Appendix E). This is supported by Bernanke’s argument – Bernanke (2013) argued that “The benefits of monetary accommodation in the advanced economics are not created in any significant way by changes in exchange rate; they come instead from the support for domestic aggregate demand in each country or region. Moreover, because stronger growth in each economy confers beneficial spillovers to trading partners, these policies are not ‘beggar-thy-neighbor’ but rather are positive-sum, ‘enrich-thy-neighbor’ actions.”

In my view, monetary policy can only do so much when other economic policies are floundering. Gustavo, Lama, and Medina (2017) find that, in the Nash equilibrium, cooperative unconventional monetary policy results in the RER adjustment, although it does not imply a “beggar-thy-neighbor.” I strongly believe that we should discuss the QQE2 why BoJ continues to adopt uncooperative unconventional monetary policy.

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[14] In the non-linear differential equation, the noise is taken into not only “input” but also “output” account.

[15] Shelton (2017) writes, “There’s no denying that one of the primary arrow of Japan’s economic strategy under Prime Minister Shinzo Abe, starting in late 2012, was to use radical quantitative easing to boost the “competitiveness” of Japan’s exports.

[16] We, the G7 Ministers and Governors, reaffirm our longstanding commitment to market determined exchange rates and to consult closely in regard to actions in foreign exchange markets. We reaffirm that our fiscal and monetary policies have been and will remain oriented towards meeting our respective domestic objectives using domestic instruments, and that we will not target exchange rates. We are agreed that excessive volatility and disorderly movements in exchange rates can have adverse implications for economic and financial stability. We will continue to consult closely on exchange markets and cooperate as appropriate. http://www.mof.go.jp/english/international_policy/convention/g7/g7_130212.pdf

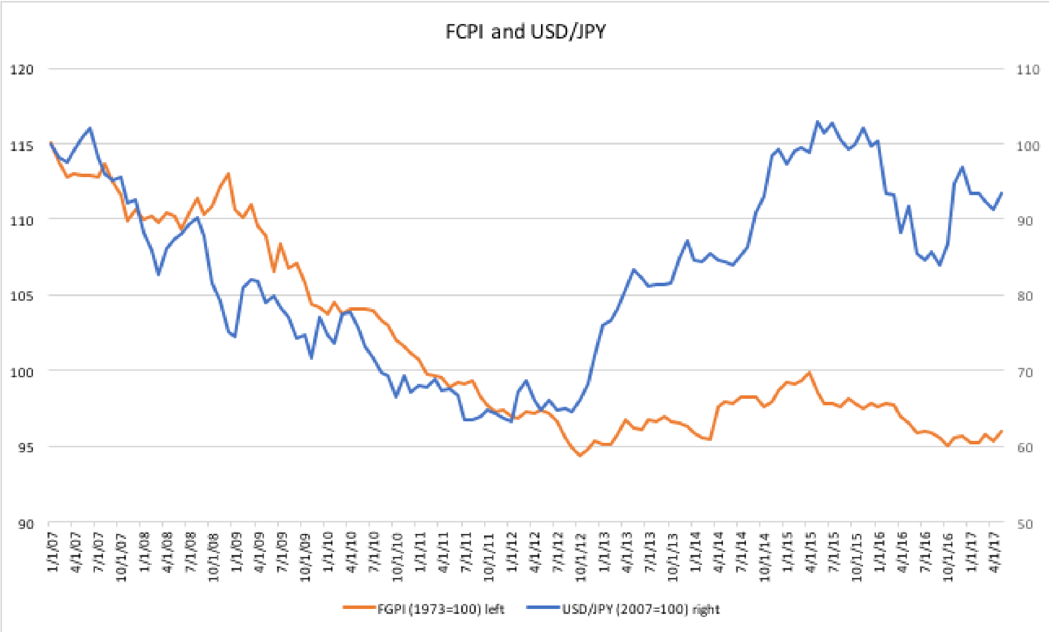


Figure 4.

Note: The graph shows the cumulated depreciation (+) or appreciation (-) of the yen against the dollar since January 2007 (=100). About 40%-yen appreciation against the dollar between 2007 and 2012 was entirely reversed following the implementation of Abenomics (April 2013).

4. Discussion: the RER cannot be hovering on the EER

I now discuss why the real exchange rate (RER) cannot remain close to the equilibrium exchange rate (EER) stably. Mr. Beryl Sprinkel, the former undersecretary of the US Treasury, had the same question. He indicates that the EER was whatever an actual market rate is at the moment. Krugman (1990) states that what advocates of some deliberate policy toward the exchange rate believe is not that there is literal disequilibrium in the market, but something more complex. Frankel and Rose (1995) and Rogoff (2002) state that economic fundamentals are weakly related to exchange rate fluctuations in the short-run, and Rogoff (1996) calls it, "it is the PPP puzzle[17]."

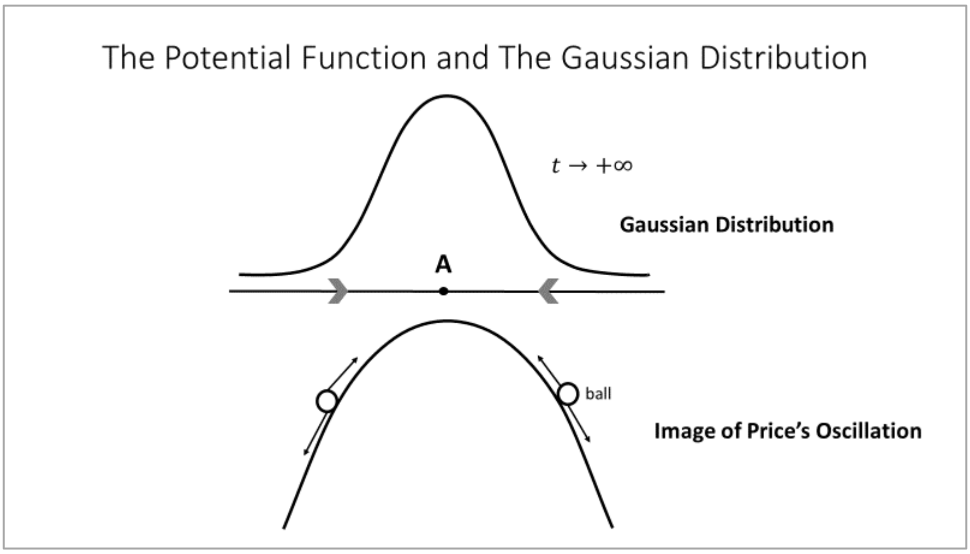
[17] There is controversy how to measure the PPP, there are controversial discussions. "More production costs in Japan are deduced, the lower the price per unit of export leads to yen's EER against the dollar strength," Kazuma (2013) said. "If Japan escapes deflation and Japanese firms come to secure proper margins on par with their U.S. counterparts, an exchange rate of 100 yen or weaker to the dollar may well become justifiable."

Why can't the RER persist in being at the equilibrium point such as the EER? There are errors between the real-time series and the NDE model time series. The error consists of a white noise. The errors between the RER and the NDE model can be depicted by a stochastic differential equation (SDE).

In Appendix D, the error of the term $\frac{d^3}{dt^3}\big(f(t)\big) + 2\frac{d^2}{dt^2}\big(f(t)\big)$with "output" is almost zero including white noise compared with$\frac{d^3}{dt^3}\big(f(t)\big)$. Accordingly, the factor of white noise is $\frac{d^3}{dt^3}\big(f(t)\big) + 2\frac{d^2}{dt^2}\big(f(t)\big)$. The SDE is:

$$\frac{d}{dt}\big(f(t)\big) = -2f(t) + \sigma\varepsilon_{t.} \qquad 4.1$$

Equation 4 -1[18]is 2nd-order function, where $\varepsilon_t$is white noise. It seems that white noise is produced by psychology, e.g., technical chart analysis, a statement of central banks or executives, news headlines, a geopolitical risk, or an option price. Figure 5 shows a potential chart for Equation 4.1. The RER converges to point A in Figure 5, which is the EER, is converged finally, but the RER is influenced by white noise. Therefore, the RER can't remain at the EER forever. Before the EER converges to point A, the market considers the next "A points" which is changed by updated fundamentals. For these reasons the RER cannot stay at the EER.



[18] Takeyuki Hida (2009) discusses about a linear additive process – the noise can be decomposed into two parts, Gaussian and compound Poisson. (See https://www.ocf.berkeley.edu/~lekheng/interviews/TakeyukiHida.pdf.)

Figure 5.

The USD/JPY exchange rate is also influenced by white noise in their model. The shifted price range wouldn't go back the EER quickly. However, the exchange rate does eventually return to the EER from a shifted price range, because the NDE is described by a 2nd-order potential function with Gaussian distribution[19].

5. Conclusion

This paper shows that the non-linear differential equation (NDE) model is useful to describe the monthly USD/JPY exchange rate time series. Additionally, I discuss 1) the linkage between monetary policy and the exchange rate, and 2) allowing difficulty to movement along the EER.

From the NDE model, the ultra-loose monetary policy by BoJ leads to a distorted currency value – excessive depreciation of the yen. Before enacting QQE 2, the BoJ should consider the impact whether or not abused monetary policy disrupts a positive harmony to the exchange rate policy.

Due to white noise produced by the errors, the real exchange rate (RER) cannot be hovering on the equilibrium exchange rate (EER). By the influence of the noise, one price range is shifted to another price range. By the noise, however, the RER doesn't deviate from a theoretical value such as the PPP because the exchange rate can be dominated by a currency policy.

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Emeritus Professor Yoshihiro Shikata, Graduate School of Mathematics at Nagoya University, and Mr. Stephen Wolff, Department of Mathematics at Rice University, provided their excellent comments.

Appendix. A

By Fourier transform, the time series removes white noise. The n-point (n =power of 2) Fourier transform of a signal $\vec{x} = [x_t]$, $t = 0, 1, \ldots, n-1$ is defined to be a sequence $\vec{X}$ of $n/2 + 1$ complex numbers $X_f, f = 0, 1, \ldots, n/2$, given by $X_f = R_f + iI_f$ in which,

$$R_f = \sum_{t=0}^{n-1} x_t \cos(2\pi f t/n)$$

and

[19] A ball in Figure 5 could not run out without changing the form of the potential function.

$$I_f = \sum_{t=0}^{n-1} x_t \sin(2\pi ft/n),\ f = 0, \dots, n/2 \quad (1)$$

where i is the imaginary unit. The signal $\vec{x}$ can be recovered by the inverse transform:

$$x_t = (R_0 + R_{n/2} \cos(\pi_t)/2 + \sum_{f=1}^{n/2-1} R_f \cos(2\pi ft/n) + \sum_{f=1}^{n/2-1} I_f \sin(2\pi ft/n),$$
$$t = 0, \dots, n-1 \quad (2)$$

To efficiently filter noise and perform reduction on data, we add a parameters m to Equation 2 to control the reduction rate. Equation 2 is then changed to Equation 3.

$$x_t = (R_0 + R_{n/(2m)} \cos(\pi_t)/2 + \sum_{f=1}^{n/(2m)-1} R_f \cos(2\pi ftm/n) + \sum_{f=1}^{n/(2m)-1} I_f \sin(2\pi ftm/n), t = 0, \dots, n/m-1 \quad (3)$$

Appendix. B

Factor analysis is a standard statistical tool. The core part of the factor analysis is to approximate the target series $f(t)$ as a linear combination

$$f(t) = A \cdot a(t) + B \cdot b(t) + \cdots \quad (1)$$

To measure the approximate degree, one uses the L^2 norm $[\![\ ]\!]$ (see Appendix B):

$$[\![f(t) - (A \cdot a(t) + B \cdot b(t) + \cdots)]\!] = \sum |f(t) - (A \cdot a(t) + B \cdot b(t) + \cdots)|^2 \quad (2)$$

Thus, factor analysis turns out to be a type of variation problem for the evaluation function $\|f(t) - (A \cdot a(t) + B \cdot b(t) + \cdots)\|$, namely, to minimize the sum

$$V(f(t), a(t), b(t), \dots : A, B, \dots) = V(A, B, \dots) = \sum |f(t) - (A \cdot a(t) + B \cdot b(t) + \cdots)|^2 \quad (3)$$

by a suitable choice of the constants $A, B, \cdots$

If I use the inner product <, > of vectors, the function above can be rewritten as

$$V(A, B, \dots) = \;< \{f(t) - (A \cdot a(t) + B \cdot b(t) + \cdots)\}, \{f(t) - (A \cdot a(t) + B \cdot b(t) + \cdots)\} > \quad (4)$$

and expanded into a quadratic form in $A, B, \dots$ as follows:

$$V(A, B, \cdots) = A^2 < a(t), A(t) > + B^2 < b(t), b(t) > + \cdots + 2AB < a(t), b(t) > + \cdots + 2A < a(t), f(t) > + 2B < b(t), f(t) > + \cdots + < f(t), f(t) > \quad (5)$$

Differentiating$V(A, B, \dots)$ in $A, B, \dots$, and setting the result equal to zero, I find that the answer to the variation problem; it is given by the simultaneous linear equations in $A, B, \dots$ with the coefficients $< a(t), A(t) >, < b(t), b(t) >, < a(t), b(t) >, < a(t), f(t) >, < b(t), f(t) >, \dots, < f(t), f(t) >$. This indicates once I choose the time series $a(t), b(t), \dots, f(t)$, then the minimal value of $V(A, B, \cdots)$ is determined automatically.

For ease of computation, I introduce an order into the polynomial algebra generated on the basis $g(t), \frac{d}{dt}g(t), \frac{d^2}{dt^2}g(t), \dots$ for a differentiable function $g(t)$, then the differentials $\frac{d}{dt}g(t)$, $\frac{d^2}{dt^2}g(t)$ ... are of order 2, 4, etc. .

Choose a basis of the algebra of order not greater than 5,

$$B(g{:}4) = \left\{1, g(t), g^2(t), \cdots, \frac{d}{dt}g(t), \frac{d}{dt}g(t) \cdot g(t), \frac{d}{dt}g(t) \cdot g^2(t), \frac{d^2}{dt^2}g(t)\right\} \quad (6)$$

as a vector space, then define

$$B(g{:}4) = B(4) - \{g(t)\} = \{\text{exclude } g(t) \text{from } B(4)\}. \quad (7)$$

For a target function $f(t)$ and for the functions in $B(f;4)$, I can minimize the variation $V(f(t),\ B(f{:}4){:} A,\ B,\ \dots)$.

Appendix. C

The L^p-norm of a function f is

$$\|f\|_p = \left(\int_{-1}^{1} |f(x)|^p dx\right)^{1/p} \quad (1)$$

This is defined for $1 \le p < \infty$ and for any measurable f. The function space $L^p[-1, 1]$ is the class of measurable functions for which the above norm is finite. The norm $\|f\|_\infty$ of a measurable function f is the *essential supremum*. Roughly speaking, this is the largest value of $|f(x)|$ if you ignore sets of measure zero. It turns out to be the limit of the norms $\|f\|_p$ as p tends to infinity. The space $L^\infty[-1,1]$ consists of those measurable functions f for which $\|f\|_\infty$ is finite. While the L^∞ norm is concerned solely with the "height" of a function, the L^p norms are instead concerned with a combination of the "height" and "width" of a function.

Particularly important among these norms is the L^2 -norm, since $L^2[-1,1]$ is a Hilbert space. This space is exceptionally rich in symmetries: It admits a wide variety of *unitary transformations*, that is, invertible linear maps T defined on $L^2[-1,1]$ such that $\|Tf\|_2 = \|f\|_2$ for every function $f \in L^2[-1,1]$ (See Gowers 2008).

Appendix. D

I show the general solution of the differential equation.

$$f(t) = \left(\frac{d^3}{dt^3}f(t)\right)^2 + 2\cdot\frac{d^3}{dt^3}f(t)\cdot\frac{d^2}{dt^2}f(t) \tag{1}$$

Putting

$$f(t) = \frac{d^3}{dt^3}f(x)\cdot\left\{\frac{d^3}{dt^3}f(t) + 2\cdot\frac{d^2}{dt^2}f(x)\right\} \tag{2}$$

$(0 \doteqdot)\frac{d^3}{dt^3}f(t) + 2\cdot\frac{d^2}{dt^2}f(t) < \frac{d^3}{dt^3}f(t)$, because of comparison of error $\frac{d^3}{dt^3}f(t)$ with a stage of "output" with a real-time series data.

Therefore

$$\frac{d^3}{dt^3}f(t) + 2\cdot\frac{d^2}{dt^2}f(t) = \varepsilon_t \tag{3}$$

Integrating equation (3) twice with respect to time yields

$$\frac{d}{dt}f(t) = -2f(t) + \varepsilon_t \tag{4}$$

Thus $f(t) = Y\cdot e^{-2t}$, by the general solution of $\frac{d}{dt}f(t) + 2f(t) = 0$.

Equation (4) is

$$\frac{d}{dt}Y\cdot e^{-2t} = \varepsilon_t \tag{5}$$

$$Y = \sum_{k=1}^{t} e^{-2k}\cdot\varepsilon_k \tag{6}$$

Therefore

$$f(t) = e^{-2t} \cdot \sum_{k=1}^{t} e^{-2k} \cdot \varepsilon_k \qquad (7)$$

Appendix. E

The Finished Goods Price Index (FGPI) is based on the Purchasing Power Parity (PPP), which compares different countries' currencies through a market "basket of goods" approach.

More precisely,

$$EX = P_1/P_2 \qquad (1)$$

where EX represents the USD/JPY exchange rate, P_1 represents a monthly change of PPI(Producer Price Index) Finished Goods in the US, and P_2 represents a monthly change of PPI in Japan. The FGPI value is 1973=100. Data are obtained from Bloomberg.

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