DYNAMICS OF STOCK MARKET, USING ISING MODEL

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Abstract. *The Ising model provides a general background to build realistic models of social interaction [1, 2]. The agents of stock market have two options buy or sell based on the neighborhood interaction, external information etc. We study the impact of interaction between agents by studying entropy, price returns and correlation between agents. The effect of average amount of money per agent on these factors and the role of herd behavior on stock market crashes also are studied*

Keywords: *Ising model, stock market, statistical physics.*

1. Introduction

Any marketplace that allows agents or traders to buy or sell financial goods such as bonds, currencies, stocks etc, is called a financial market. When the trading involves the buying or selling of shares in a publicly traded company, that financial market is more commonly referred to as a stock market, where a stock is essentially a share of ownership of that company [3]. For many years researchers were interested in exploring the reasons for crashes and bubbles in stock markets, in order to prevent crashes and eventually get profit from bubbles (the brokers overvalue some assets; these stocks are bought in order to resell later on, not for their realistic value). Nevertheless, the Efficient Market Hypothesis [4] which is a popular model in economics has not been able to explain market details and even more important, it could not explain market crashes. After that new models were introduced to understand the human influence upon financial decision making [5, 6].

In reality, markets are networks of influence i.e., people talk to each other, and look at each other for decisions. This means that in order to have a complete picture of the system, internal interaction has to be considered.

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The subset of economic decision problems has a structure identical to that of classical thermodynamics. Universality in statistical physics says that different materials behave exactly the same near their respective critical points. We can find an analogy of this in economic systems also. Stock prices respond to fluctuation in demand just as magnetization of an interacting spin system responds to fluctuations in the magnetic field [7-10].

In the stock market the price changes depending on demand and supply. When there is excess demand (or number of buyers exceeds the number of sellers), the price rises and when there is excess supply (or number of sellers exceeds number of buyers) the price falls. Prices can be seen as information. As such, price movement can be interpreted as information moving between agents in the market. When large numbers of agents decide to sell, an imbalance is created and this is balanced by a lowering of price which results in a crash. Such crashes are not very frequent but happen. Then certain agents overvalue some assets and buy them to resell which is called speculative bubbles. This situation can be compared with phase transition problems in physics. For such physical systems the inter particle interaction cannot be removed, and hence energy levels of the total system cannot be related to energy levels of individual constituents [11]. In order to study cooperative phenomena like this we use different models [11]. Among them Ising model is considered as the effective model for systems having phase transitions. The investors in stock market expect some income when they invest their money. This may be in the form of profits from trading of shares or dividends received. This is called price returns which are bound to be affected by various risks in the market such as political conditions, economic crisis, natural disasters, international oil prices, inflation effects etc [12]. Hence study of volatility of price returns is an interesting field. Correlation among the traders also affects the stock market crashes. It can be used to predict crashes in market. One of the factors which affect the prices in stock market is the changes in the exchange rate of money [13]. As we know such changes are reflected in GDP or the average amount of money per individual. In section 3 we study the influence of average amount of money per agent on the order, price returns and correlation.

2. One dimensional Ising model in Physics

The Ising model, introduced initially as a mathematical model of ferromagnetism in statistical mechanics, is now part of the common culture of physics as the simplest representation of interacting elements with a finite number of possible states. The model consists of a large number of magnetic moments (or spins) connected by links within a graph, network or grid. In the simplest version, the spins can only take two values (±1) which represent the direction in which they point (up or down). Each spin interacts with its direct neighbors, tending to align together in a common direction, while the temperature tends to make the spin orientations random.

In one dimensional Ising model we consider N lattice sites arranged as a ring. Associated with each lattice there is a spin variable $\sigma_i (i = 1,2,3,\ldots\ldots, N)$ which can take either +1 or –1. If $\sigma_i = +1$ the i^{th} number is said to have spin up and $\sigma_i = -1,$ it is said to be spin down. In order to avoid end effects, the periodic boundary condition is assumed [11, 14].

$$\sigma_{N+1} = \sigma_1.$$

The Hamiltonian is given by

$$E(\sigma_i) = -J\sum_{i=1}^{N}\sigma_i\sigma_{i+1} - \mu B\sum_{i=1}^{N}\sigma_i. \tag{1.1}$$

We have N particle partition function

$$Q_N = \sum_{overallstates} e^{(-\beta E(\sigma_i))} \tag{1.2}$$

$$Q_N = \sum_{\sigma_1=\pm 1}\ \sum_{\sigma_2=\pm 1}\ \sum_{\sigma_3=\pm 1} \ldots\ldots e^{\left(-\beta\left[-J\sum_{i=1}^{N}\sigma_i\sigma_{i+1} - \frac{\mu B}{2}\sum_{i=1}^{N}(\sigma_i + \sigma_{i+1})\right]\right)}. \tag{1.3}$$

3. Ising model of Stock market

There is a long tradition of using the Ising model and its extension to represent social interaction and organizations. Among the various statistical models proposed the Ising model and its variants [15-17] played an important role in understanding the origin of the so-called stylized facts in real financial markets. A large set of economic models can be mapped onto various versions of the Ising model to account for social influence in individual decisions [18]. The Ising model is indeed one of the simplest models describing the competition between the ordering force of imitation or contagion and the disordering impact of private information.

The Ising model is used by researchers in Econophysics to represent opinions in stock market. The simplest one dimensional Ising model can be effectively used to represent the traders' decision to buy or sell which determines the overall behavior of the market. There are two forces that influence the decision of agents – the opinions of nearest neighbors and news from external agencies. Each agent has a tendency to imitate his neighbor which creates order in the market, but the impact of external influence will try to introduce disorder. When imitation dominates there is a tendency of complete buying or selling state. The Ising model can also be used to analyze the order prevailing in the system by the use of entropy (S) parameter. The net magnetization (m) in physics corresponds to the price returns. This is because if the number of buyers is larger than that of the sellers i.e., if m is positive, the price increases definitely. On the other way if the number of sellers is more, m is negative and price decreases. The average amount of money per agent is taken as economic temperature (T). When this is low the imitation tendency is large and as T increases, agents decide independently and at a particular temperature there is complete disorder in the market, the market stabilizes slowly. This can be studied using the temperature dependence of entropy. Effect of temperature and interaction among agents on price returns also can be studied. The correlation length is a measure of order prevailing in the market. Its nature with change in economic temperature and interaction strength is studied.

Theory of stock market

In our model we consider N agents placed on a one dimensional lattice with periodic boundary conditions same as used in Ising model. This geometry is taken in order to have simple and specific way of determining who is interacting with whom. The amount of money M can be written as

$$M = -C_0 \sum_{i,j} \lambda_i \lambda_j - C_1 \sum_i \lambda_i \qquad (2.1)$$

C_0 is the factor determining the interaction between agents and C_1 quantifies the impact of the external news on the decision of agent i. The parameters λ_i, λ_j can take two states +1 (buy state) and –1 (sell state). We have the partition function,

$$Q_N = \sum_{\text{over all states}} e^{-M(\lambda)/T}. \qquad (2.2)$$

Here T is the economic temperature which is average amount of money per agent [19,20]. Substituting (2.1) in (2.2)

$$Q_N(T,C_1)=\left[e^{\frac{C_0}{T}}\cos h\left(\frac{C_1}{T}\right)+\left[e^{\frac{-2C_0}{T}}+e^{\frac{2C_0}{T}}\sin h^2\left(\frac{C_1}{T}\right)\right]^{1/2}\right]^N . \qquad (2.3)$$

'Free money' equivalent to free energy [21] is defined as

$$A=<M>-TS=-T\ \text{In}\ Q(T) \qquad (2.4)$$

where <M> is the average value of money, so that

$$S=-\left(\frac{\partial A}{\partial T}\right)_{<M>} \qquad (2.5)$$

where S is the economic entropy which is related with the disorder in the market similar to the entropy in physical systems from which all thermodynamic information can be obtained. From Q_N, we get free money and economic entropy as,

$$A=-NC_0-NT\ \text{In}\left[\cos h\left(\frac{C_1}{T}\right)+\sqrt{e^{\frac{-4C_0}{T}}+\sin h^2\left(\frac{C_1}{T}\right)}\right] \qquad (2.6)$$

$$S=\frac{-N}{X}\left[\frac{C_1}{T}\sin h\left(\frac{C_1}{T}\right)-\frac{\frac{2C_0}{T}e^{\frac{-4C_0}{T}}}{\left[\cos h\left(\frac{C_1}{T}\right)+X\right]}\right]+N\ \text{In}\left[\cos h\left(\frac{C_1}{T}\right)+X\right].$$

Where $$X=\sqrt{e^{\frac{-4C_0}{T}}+\sin h^2\left(\frac{C_1}{T}\right)} . \qquad (2.7)$$

The entropy is a measure of disorder in the market. Its variation with average money per agent is plotted. The value of S is found to increase sharply with increase in average money per agent initially, then after a particular value of average money per agent it decreases and the system attains a complete order. It is interesting to see that the interaction between the agents also influences order in the market. At a constant average money per agent entropy is plotted against C_0. It is clear that the entropy is minimum when there is 100% interaction between agents.

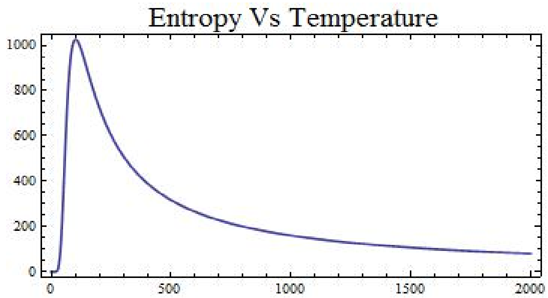

Figure 1

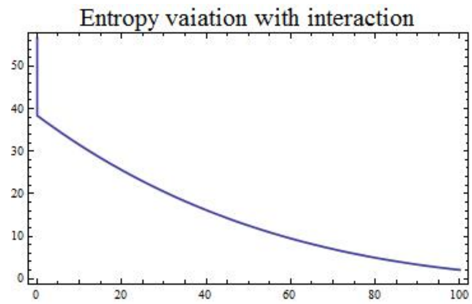

Figure 2

Thus entropy is a useful tool to measure order in a stock market. Net magnetization is the price returns in the stock market and is given by

$$m = \frac{N \sin h \frac{C_1}{T}}{X}. \tag{2.8}$$

The variation of m with economic temperature and interaction is plotted in figures 3 and 4.

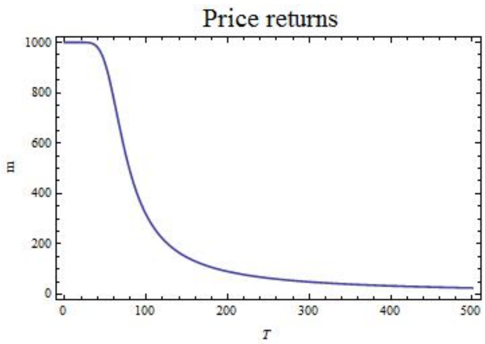

Figure 3

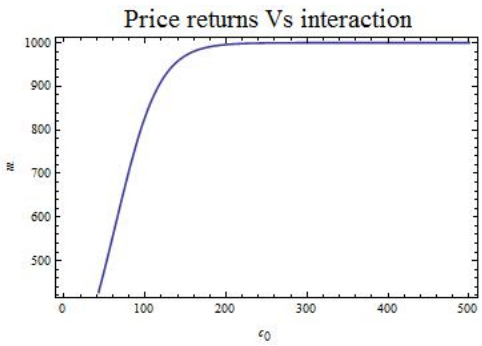

Figure 4

From the graph it is clear that at a particular temperature or at the time of market crash the price returns suddenly drop. The price returns can be increased by interaction among the agents. In a market with short range interaction there is no ordered state. In order to study the agent-agent correlation the effect of external news is not considered. It is determined by the correlation function $g(r)$ [23] where r is the distance between agents in the chain.

We have $g(r) = e^{-\frac{r}{\xi}}$ where ξ is the correlation length. For the market it is obtained as

$$\zeta = \frac{1}{\text{In cot } h\left(\frac{C_0}{T}\right)}. \tag{2.9}$$

Correlation length is plotted against temperature and interaction strength in figures 5 and 6.

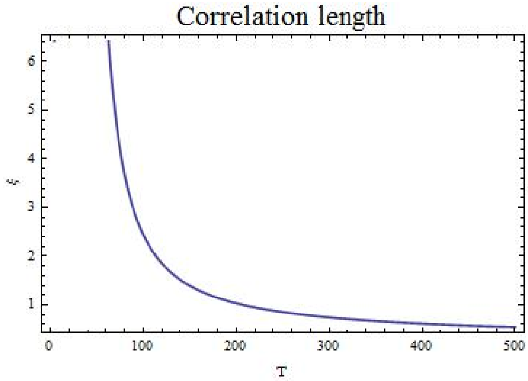

Figure 5

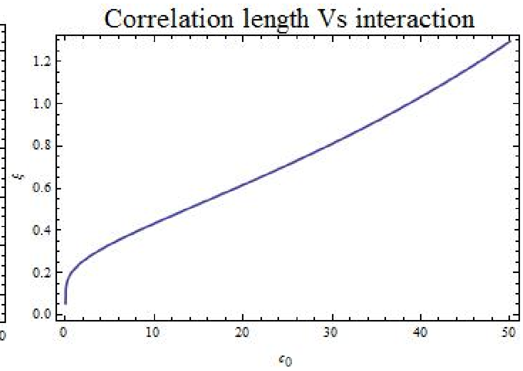

Figure 6

It is clear that correlation length is maximum at the time of crash. Maximum correlation length indicate long range interaction which leads to ordering which is drastic situation with respect to the stock market i.e., agents collectively decide to either buy or sell which affects the price of stocks as mentioned earlier. The agent interaction plays an important role in this that correlation length increases with interaction as shown in figure.

4. Conclusion

We have used Ising model to study the dynamics of stock market. We have used entropy, tock returns and correlation length to study the characteristics of stock market during crash. The effect of interaction among agents also is studied. It is found that correlation length increases with interaction which results in herding in the stock market which can lead to crash. Ising model is an effective model to study financial systems in which decision making plays an important role.

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