A COMBINATORIAL PROBLEM FOR POSSIBLE STATES ON THE ARRIVAL LINE FOR *N* COMPETITOR RUNNERS

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Abstract*. In a very small* ε*-time interval, several runners could occupy the same place on the arrival line. This possibility creates a combinatorial problem and a statistical one. The work gives the solution for both problems.*

Keywords: *Arrival line, non-nominal state, nominal state, partial feequency, final frequency, algorithm of arrival line.*

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1. Problem formulation. Special notations

A number of n competitors run to reach on the arrival line S. Each runner has his special sign (on the shirt): A, B, C, D etc.

Work hipothesis. We denote by t_A the time for the runner A and t_B the time for B. If $|t_A - t_B| \in [0, \varepsilon]$, uhere $\varepsilon > 0$ is very small (in

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milli-secomds), then both runners ocupy the same place: Place1= I or Place2 = II or Place3 = III etc.

The total number of runners arriving on a special place is denoted by

I II III IV etc.

$i \quad j \quad k \quad l \cdots\cdots\cdots\cdots p; \quad i \neq 0,$ where

$i, j, k, l, \cdots, p \in \{0, 1, 2, 3, 4, \cdots, n\}$ and $i + j + k + l + \cdots + p = n.$

Definition 1. The set $(i, j, k, \cdots, p)$ is called **non-nominal state.** By i is known only the total name of runners ocuppying the place I; by j is known only the total name of runners ocuppying the place II etc.

The value $N = N(n)$ is the **total number of non-nominal states**.

Example 1. $n = 5,$ (3, 1, 1, 0, 0) is a non-nominal state. The set (3, 1, 0, 1, 0) is not a correctly non-nominal state.

Proposition 1. $N = N(n) = 2^{n-1}, \; n \geq 1$. (1)

The variable $s = \overline{1, N}$ counts the non-nominal states.

$n = 4,$ $(AB, C, D, -),$ $(A, BC, D, -),$ $(B, C, AD, -)$ are examples of nominal states.

$T = T(n; s)$ is the total number of nominal states generated by the non-nominal state $s, 1 \leq s \leq N(n).$

$S(n)$ = is the total number of nominal states.

$Place1 = I,$ $Place2 = II,$ $Place3 = III$ etc. are the positions on the arival line.

$FP(n; s; Name, PlaceX)$ = is the **partial frequency** = the number of favorable cases for a runner to occupy the position $PlaceX$ in the **final classification,** for the non-nominal state $s, 1 \leq s \leq N(n).$

$FF(n; Nume, PlaceX)$ = is the **final frecuency** = the number of favorable cases for a nominated runner (A, B, C, D etc.) to occup the position $PlaceX$ in the final classification.

Example 2. $FF(n = 4; A, I),$ $FF(n = 5; A, I),$ $FF(n = 5; B, II).$

In an informatics model, the values FP and FP could be used as the functions having 4 or 3 variables, respectively.

2. Example of non-nominal staes and the total number $N(n) = 2^{n-1}$

Example 3.

$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
I	I II	I II III	I II III IV	I II III IV V
(1)	(2, 0)	(3, 0, 0)	(4, 0, 0, 0)	(5, 0, 0, 0, 0)
$N=1$	(1, 1)	(2, 1, 0)	(3, 1, 0 0)	(4, 1, 0, 0, 0)
	$N=2$	(1, 2, 0)	(2, 2, 0, 0)	(3, 2, 0, 0, 0)
		(1, 1, 1)	(2, 1, 1, 0)	(3, 1, 1, 0, 0)
		$N=4$	(1, 3, 0, 0)	(2, 3, 0, 0, 0)
			(1, 2, 1, 0)	(2, 2, 1, 0, 0)
			(1, 1, 2, 0)	(2, 1, 2, 0, 0)
			(1, 1, 1, 1)	(2, 1, 1, 1, 0)
			$N=8$	(1, 4, 0, 0, 0)
				(1, 3, 1, 0, 0)
				(1, 2, 2, 0, 0)
				(1, 2, 1, 1, 0)
				(1, 1, 3, 0, 0)
				(1, 1, 2, 1, 0)
				(1, 1, 1, 2, 0)
				(1, 1, 1, 1, 1)
				$N=16$.

Remark. At the concrtuction of all non-nominal states one recommends that the string for the position I to be a decreasing string. The same rule is for position II etc.

3. The construction of the set of all nominal states on the arrival line. The algorithm of arrival line. The main problem of arrival line

Definition 2. Each non-nominal state $s, 1 \le s \le N$ generates the set having $T = T(n;s)$ **nominale states** m where each runner is known by its identification sign.

We denote by $S(n)$ **the total number of nominal states** on arrival line S. Each runner is known by its identification sign.

There exists the relation $S(n) = \sum_{s=1}^{N} T(n;s)$ (2)

It remains the problem to compute the value $T(n;s)$, for $1 \le s \le N$.

Proposition 2. Let s be a non-nominal state $s, 1 \le s \le N$. Then we have the formula (3) illustrated for $n=4$

$$s, (i, j, k, l),\ i+j+k+l=4,\ T(n;s) = C_n^i C_{n-i}^j C_{n-i-j}^k C_{n-i-j-k}^l. \qquad (3)$$

The generalisation is immediately.

Example 4. $n=4$, $s=5$, $(i, j, k, l) = (1, 2, 1, 0)$, $i+j+k+l=4$.

$$T(4;5) = C_n^i C_{n-i}^j C_{n-i-j}^k = C_4^1 C_{4-1}^2 C_{4-1-2}^1 = C_4^1 C_3^2 C_1^1 = 4 \cdot 3 \cdot 1 = 12.$$

Example 5. $n=5$, $s=13$, $(i, j, k, l, p) = (1, 2, 1, 1, 0)$, $i+j+k+l=5$.

$$T(5;13) = C_n^i C_{n-i}^j C_{n-i-j}^k C_{n-i-j-k}^l = C_5^1 C_{5-1}^2 C_{5-1-2}^1 C_{5-1-2-1}^1 = C_5^1 C_4^2 C_2^1 C_1^1 = 60.$$

Remark. The total number of nominal states $S(n)$ could be calculated by two methods.

Method 1. For a small value on n we construct the whole set and (directly) count the total number $S(n)$.

Method 2. We generate the set having $T = T(n;s)$ non-nominal states and apply the formulas (3) and (2).

The algorithm of arrival line

Step 1. Find the total number $N = N(n) = 2^{n-1}$ of al non-nominal states.

Step 2. Construct the **string of non-nominal states** $(i, j, k, \cdots, p)$, for $s = \overline{1, N}$.

Step 3. For each non-nominal state $s, 1 \le s \le N$ we construct **the set of all nominal states.** Each state has $T = T(n;s)$ nominal states, where each runner is known by its identidication sign or number.

Step 4. We save all the natural numbers $T = T(n;s)$, $s = \overline{1, N}$.

Step 5. One calculates the total number of nominal states $S(n) = \sum_{s=1}^{N} T(n;s)$.

4. Numerical example for $n=5$

Example 6. For $n=5$ the runners are A, B, C, D, E.

a) Find $N = N(n)$ and construct all non-nominal states.

b) Construct nominal staes and find $T = T(n;s)$, for any $1 \le s \le N$.

c) Find the total number of nominal states $S(n) = \sum_{s=1}^{N} T(n;s)$.

Soluție. Aplicăm algoritmul liniei de sosire.

a) $n = 5$, $i, j, k, l, p \in \{0,1,2,3,4,5\}$; find the string of non-nominal states.

$s = 1$	(5, 0, 0, 0, 0)
$s = 2$	(4, 1, 0, 0, 0)
$s = 3$	(3, 2, 0, 0, 0)
$s = 4$	(3, 1, 1, 0, 0)
$s = 5$	(2, 3, 0, 0, 0)
$s = 6$	(2, 2, 1, 0, 0)
$s = 7$	(2, 1, 2, 0, 0)
$s = 8$	(2, 1, 1, 1, 0)
$s = 9$	(1, 4, 0, 0, 0)
$s = 10$	(1, 3, 1, 0, 0)
$s = 11$	(1, 2, 2, 0, 0)
$s = 12$	(1, 2, 1, 1, 0)
$s = 13$	(1, 1, 3, 0, 0)
$s = 14$	(1, 1, 2, 1, 0)
$s = 15$	(1, 1, 1, 2, 0)
$s = 16$	(1, 1, 1, 1, 1).

b) Construct the set of nominal states on arrival line, for $n = 5$.

Place1	Place2	Place3	Place4	Place5	(i, j, k, l, p); $i + j + k + l + p = 5$
ABCDE	–	–	–	–	$s = 1$ (5,0,0,0,0)
ABCD	*E*	–	–	–	$s = 2$ (4,1,0,0,0)
ABCE	*D*	–	–	–	
ABDE	*C*	–	–	–	
ACDE	*B*	–	–	–	
BCDE	*A*	–	–	–	

Remark. If we invert the columns Place1 and Place2 we obtain the states of (1,4,0,0,0).

ABC	*DE*	–	–	–	$s = 3$ (3,2,0,0,0)
ABD	*CE*	–	–	–	
ABE	*CD*	–	–	–	
ACD	*BE*	–	–	–	
ACE	*BD*	–	–	–	

ADE	*BC*	–	–	–	
BCD	*AE*	–	–	–	
BCE	*AD*	–	–	–	
BDE	*AC*	–	–	–	
CDE	*AB*	–	–	–	

Remark. If we invert the columns Place1 and Place2 we obtain the states of (2,3,0,0,0).

ABC	*D*	*E*	–	–	$s = 4$ (3,1,1,0,0)
ABC	*E*	*D*	–	–	
ABD	*C*	*E*	–	–	
ABD	*E*	*C*	–	–	
ABE	*C*	*D*	–	–	
ABE	*D*	*C*	–	–	
ACD	*B*	*E*	–	–	
ACD	*E*	*B*	–	–	
ACE	*B*	*D*	–	–	
ACE	*D*	*B*	–	–	
ADE	*B*	*C*	–	–	
ACE	*C*	*B*	–	–	
BCD	*A*	*E*	–	–	
BCD	*E*	*A*	–	–	
BCE	*A*	*D*	–	–	
BCE	*D*	*A*	–	–	
BDE	*A*	*C*	–	–	
BDE	*C*	*A*	–	–	
CDE	*A*	*B*	–	–	
CDE	*B*	*A*	–	–	

Remark. If we invert the columns Place1 and Place2 we obtain the states of (1,3,1,0,0).

This artificial method also could be applied for other non-nominal state, in oder to obtain the nominal states.

AB	*CDE*	–	–	–	$s = 5$ (2,3,0,0,0)
AC	*BDE*	–	–	–	
AD	*BCE*	–	–	–	
AE	*BCD*	–	–	–	
BC	*ADE*	–	–	–	
BD	*ACE*	–	–	–	
BE	*ACD*	–	–	–	

CD	ABE	–	–	–
CE	ABD	–	–	–
DE	ABC	–	–	–

$s=6$ $(2,2,1,0,0)$, $(Place1, Placec2, Place3, Place4, Place5)$
$(AB, CD, E,-,-)$, $(AB, CE, D,-,-)$, $(AB, DE, C,-,-)$
$(AC, BD, E,-,-)$, $(AC, BE, D,-,-)$, $(AC, DE, B,-,-)$
$(AD, BC, E,-,-)$, $(AD, BE, C,-,-)$, $(AD, CE, B,-,-)$
$(AE, BC, D,-,-)$, $(AE, BD, C,-,-)$, $(AE, CD, E,-,-)$
$(BC, AD, E,-,-)$, $(BC, AE, D,-,-)$, $(BC, DE, A,-,-)$
$(BD, AC, E,-,-)$, $(BD, AE, C,-,-)$, $(BD, CE, A,-,-)$
$(BE, AC, D,-,-)$, $(BE, AD, C,-,-)$, $(BE, CD, A,-,-)$
$(CD, AB, E,-,-)$, $(CD, AE, B,-,-)$, $(CD, BE, A,-,-)$
$(CE, AB, D,-,-)$, $(CE, AD, B,-,-)$, $(CE, BD, A,-,-)$
$(DE, AC, B,-,-)$, $(DE, AB, C,-,-)$, $(DE, BC, A,-,-)$.

$s=7$ $(2,1,2,0,0)$, $(Place1, Placec2, Place3, Place4, Place5)$
$(AB, E, CD,-,-)$, $(AB, D, CE,-,-)$, $(AB, C, DE,-,-)$
$(AC, E, BD,-,-)$, $(AC, D, BE,-,-)$, $(AC, B, DE,-,-)$
$(AD, E, BC,-,-)$, $(AD, C, BE,-,-)$, $(AD, B, CE,-,-)$
$(AE, D, BC,-,-)$, $(AE, C, BD,-,-)$, $(AE, E, CD,-,-)$
$(BC, E, AD,-,-)$, $(BC, D, AE,-,-)$, $(BC, AE, DE,-,-)$
$(BD, E, AC,-,-)$, $(BD, C, AE,-,-)$, $(BD, A, CE,-,-)$
$(BE, D, AC,-,-)$, $(BE, C, AD,-,-)$, $(BE, AD, CD,-,-)$
$(CD, E, AB,-,-)$, $(CD, B, AE,-,-)$, $(CD, A, BE,-,-)$
$(CE, D, AB,-,-)$, $(CE, B, AD,-,-)$, $(CE, A, BD,-,-)$
$(DE, B, AC,-,-)$, $(DE, C, AB,-,-)$, $(DE, A, BC,-,-)$.

$s=8$ (2, 1, 1, 1, 0). We find 60 nominal states.

$s=9$ $(1,4,0,0,0)$, $(Place1, Placec2, Place3, Place4, Place5)$
$(A, BCDE,-,-,-)$, $(B, ACDE,-,-,-)$, $(C, ABDE,-,-,-)$
$(D, ABCE,-,-,-)$, $(E, ABCD,-,-,-)$ and so on.

There exists a great number of nominal states, namely, $S(n=5)$ is a big number.

c) We compute the value of $S(n)=S(n=5)$, representing the total number of nominal states by two formulas.

We generate the set with $T=T(n;s)$ nominal states and apply the formulas (2) and (3).

$S(n)=\sum_{s=1}^{N} T(n;s),\ N=N(5)=16,$

$n=5,\ (i,j,k,l,p),\ i+j+k+l+p=5.$

$T(n;s)=C_n^i C_{n-i}^j C_{n-i-j}^k C_{n-i-j-k}^l C_{n-i-j-k-l}^p,\ 1\le s\le N$

$T(5;1)=C_5^5=1$	$T(5;2)=C_5^4 C_1^1=5$	$T(5;3)=10$
$T(5;4)=20$	$T(5;5)=10$	$T(5;6)=30$
$T(5;7)=30$	$T(5;8)=60$	$T(5;9)=5$
$T(5;10)=20$	$T(5;11)=60$	$T(5;12)=20$
$T(5;13)=30$	$T(5;14)=60$	$T(5;15)=60$
$T(5;16)=5!=120.$		

$S(5)=\sum_{s=1}^{16} T(5;s)=541$ nominal states.

5. The number of favorable cases for a place occupied in final classification. Computation formula for partial frequency

We recall two notions presented in section 1.

$FP(n;s;Name,PlaceX)$ = is the **partial frequency** = the number of favorable cases for a runner to occupy the position $PlaceX$ in the **final classification,** for the non-nominal state $s, 1\le s\le N(n)$.

$FF(n;Nume,PlaceX)$ = is the **final frecuency** = the number of favorable cases for a nominated runner (A, B, C, D etc.) to occup the position $PlaceX$ in the final classification.

Example 7. $FP(n=4;s=3;A,Loc1)$, or better $FP(n=4;s=3;A,I)$,

$FP(n=5;s=3;A,I)$, $FP(n=5;s=5;A,I)$,

$FP(n=5;s=3;B,Loc2)$, or better $FP(n=5;s=3;B,II)$ etc.

There are two methods to compute partial frewuencu $FP(n;s;Name,PlaceX)$.

Method 1. (direct method; direct counting). We generate all nominal states and count their total number, for a nominated runner (*A* or *B* etc.) and a nominated place (I or II etc.).

Metoda 2. (Computation formula for *FP*).

For $Place1$ and the state *s* having the form $(n,0,0,\cdots,0)$ the partial frequency is $FP(n;s;Name,Place1)=C_n^n=1$.

For $Place1$ and, for example, the state s having the form $(i, j, k.l, 0)$ we use the formula $FP(n; s; Name, Place1) = C_{n-1}^{j} C_{n-1-j}^{k} C_{n-1-j-k}^{l}$. (4)

Proposition 3. There are many cases to use the computation formulas for $Place2$, $Place3$ etc.

Case 1. We take $T = T(n; s)$ with formula (3) and the state $(i, j, 0.0, 0)$, with $i \neq j$. Then

$FP(n; s; Name, Place2) = T - FP(n; s; Name, Place1)$ (diference's method).

Case 2. We take $T = T(n; s)$ with (3) and the state starea $(i, i, 0.0, 0)$ having two equal values $i = i$. Then

$FP(n; s; Name, Place2) = FP(n; s; Name, Place1)$ (the mathod of equal values).

Case 3. We take $T = T(n; s)$ with formula (3) and the state $(i, j = 3 \cdot i, k = 2 \cdot i, 0, 0)$. Then the computation is based on the proportional method

$FP(n; s; Name, Place2) = \frac{j}{i} \cdot FP(n; s; Name, Place1)$ (proportional method)

$FP(n; s; Name, Place3) = \frac{k}{i} \cdot FP(n; s; Name, Place1)$ (proportional method).

Case 4. We take $T = T(n; s)$ with formula (3) and the state $(i, j, i, 0, 0)$. Then

$FP(n; s; Name, Place3) = FP(n; s; Name, Place1)$ (equal values).

$FP(n; s; Name, LoPlace2) =$

$= T - [\, FP(n; s; Name, Place1) + FP(n; s; Name, Place3)]$.

Now we have to compute the value $FF(n; Name, PlaceX)$ for a nominated runner, with the position *Place* in final classification.

Example 8. $FF(n = 4; A, I)$, $FF(n = 5; A, I)$, $FF(n = 5; B, II)$.

We put together all partial value FP

$$FF(n; Name, Place1) = \sum_{s=1}^{N} FP(n; s; Name, Place1) \quad (5)$$

$$FF(n; Name, Place2) = \sum_{s=1}^{N} FP(n; s; Name, Place2)$$

$$FF(n; Name, Place3) = \sum_{s=1}^{N} FP(n; s; Name, Place3) \text{ etc.}$$

6. The complete problem of arrival line

Let n be the number of competitors running to ocuppy a position on arrival line.

Step 1. Find **the total number** $N = N(n) = 2^{n-1}$ for all non-nominal staes.

Step 2. Construct the **string of non-nominal states** $(i, j, k, \cdots, p)$, for $s = 1, N$.

Step 3. For each non-nominal state $s, 1 \le s \le N$ construct **the set of nominal states**.

Step 4. Compute **the partial frequency** FP for Place1, by formula (4).

Step 5. Compute **the partial frequency** FP for Place2, Place2 etc. (proposition 3).

Step 6. Compute **the total number of favorable cases** FF for each runner, in final classification.

Step 7. Arrange (registration) the obtained numerical data s, FP, FF and $S(n)$ in a **centralisation table** or summary table.

Step 8. Make **the proof of correctitude** for all computations.

Step 9. Attach **the random** variables $X1$ for $Place1 = I$, $X2$ for $Place2 = II$, $X3$ for $Place3 = III$ etc.

Step 10. Make some **statistical computations** for random variables $X1$, $X2$, $X3$ etc.

7. The random variables attached to final clasiffication. The probability of each place on arrival line

Let n be the runners A, B, C, D, E, F etc. reaching on arrival line.

From centalization table we know the total number of favorable cases $FF(n; Name, PlaceX)$ for each runner in final classification. We simplify the writing and denote $FF(n; Name, Place1) = FFnI$, $FF(n; Name, Place2) = FFnII$, $FF(n; Name, Place3) = FFnIII$ erc.

The occupied places could be $Place1$, $Place2$, $\cdots$, $Placen$.

The score rule. Each runner occupiyng the position $PlaceX$ gives a score (a number of points). The score is at choice of organizors.

For *Place1* the value of score is α points. For example $\alpha = 100$ points.

For *Place2* the value of score is $\alpha/2$ points or $2\alpha/3$ points etc.

For *Place3* the value of score is $\alpha/3$ points or $2\alpha/3$ points etc.

We define **the discrete random variable** in a classical form (manner) and we take $\alpha = 100$.

$$X = \begin{pmatrix} x_1 & x_2 & x_3 \cdots\cdots x_n \\ p_1 & p_2 & p_3 \cdots\cdots p_n \end{pmatrix}, \; 0 \le p_i \le 1, \; p_1 + p_2 + \cdots p_n = 1$$

$$Xn = \begin{pmatrix} \frac{100}{1} & \frac{100}{2} & \frac{100}{3} \cdots\cdots \frac{100}{n} \\ \frac{FFI}{S(n)} & \frac{FFII}{S(n)} & \frac{FFIII}{S(n)} \cdots \frac{FFn}{S(n\_} \end{pmatrix}, \; FFI + FFII + \cdots + FFn = S(n) \quad (6)$$

for any natural number $n \ge 1$.

The random variable Xn defines **the random variation of score,** on arrival line. This random variable gives the possibility to execute a lot of statistical compuations [NP] like mean value $M)X) = m,$ variance $D^2(X) = \sigma^2$, standard deviation $D(X) = \sigma$ non-centered moments $M_k(X)$, centered moments $m_k(X)$, generating function $g(t)$ characteristic function $c(t)$ etc.

Here we mention only tha formulas for mean value and variance $M(X) = x_1 p_1 + x_2 p_2 + \cdots x_n p_n$, $D^2(X) = M(X^2) - [M(X)]^2$.

8. Numerical examples with statistical computations

E**xample 9.** Let $n = 5$ be the number of runners A, B, C, D, E. $N = N(n) = 2^{n-1}$.

a) Compute the partial frequency FP of favorable cases $FP(n; s; Nume, LocX)$ for each runner and any state $s, 1 \le s \le N(n)$.

b) Compute the final frequency FF of all favorable cases $FF(n; Nume, LocX)$ for each runner in final classification.

c) Make the correctitude verification.

Solution. a) WE find the partial frequency of favorable cases $FP(n; s; Nume, LocX)$, for $n = 5$ runners A, B, C, D, E.

Step 1. Constrct al non-nominal states

$s=1$	(5, 0, 0, 0, 0)
$s=2$	(4, 1, 0, 0, 0)
$s=3$	(3, 2, 0, 0, 0)
$s=4$	(3, 1, 1, 0, 0)
$s=5$	(2, 3, 0, 0, 0)
$s=6$	(2, 2, 1, 0, 0)
$s=7$	(2, 1, 2, 0, 0)
$s=8$	(2, 1, 1, 1, 0)
$s=9$	(1, 4, 0, 0, 0)
$s=10$	(1, 3, 1, 0, 0)
$s=11$	(1, 2, 2, 0, 0)
$s=12$	(1, 2, 1, 1, 0)
$s=13$	(1, 1, 3, 0, 0)
$s=14$	(1, 1, 2, 1, 0)
$s=15$	(1, 1, 1, 2, 0)
$s=16$	(1, 1, 1, 1, 1) $N(5)=2^4=16.$

Step 2. For $n \leq 4$ (n with a small value) we concstruct all nominal states on arrival line for given n.

Step 3. Compute all partial frequency for Place1, with formula (4).

Method 1. The direct method is difficult to use for $n \geq 5$.

Method 2. Wee use formula (4) for Place1.

$s=1,\ (5,0,0,0,0),\ FP(n=5; s=1; A, I) = C_5^5 = 1.$

$s=2,\ (4,1,0,0,0),\ FP(n=5; s=2; A, I) = C_4^1 = 4.$

$s=3,\ (3,2,0,0,0),\ FP(n=5; s=3; A, I) = C_4^2 = 6.$

$s=4,\ (3,1,1,0,0),\ FP(n=5; s=4; A, I) = C_4^1 C_3^1 = 12.$

$s=5,\ (2,3,0,0,0),\ FP(n=5; s=5; A, I) = C_4^3 = 4.$

$s=6,\ (2,2,1,0,0),\ FP(n=5; s=6; A, I) = C_4^2 C_2^1 = 12.$

$s=7,\ (2,1,2,0,0),\ FP(n=5; s=7; A, I) = C_4^1 C_3^2 = 12.$

$s = 8$, $(2,1,1,1,0)$, $FP(n = 5; s = 8; A, I) = C_4^1 C_3^1 C_2^1 = 24$.

$s = 9$, $(1,4,0,0,0)$, $FP(n = 5; s = 9; A, I) = C_4^4 = 1$.

$s = 10$, $(1,3,1,0,0)$, $FP(n = 5; s = 10; A, I) = C_4^3 C_1^1 = 4$.

$s = 11$, $(1,2,2,0,0)$, $FP(n = 5; s = 11; A, I) = C_4^2 C_2^2 = 6$.

$s = 12$, $(1,2,1,1,0)$, $FP(n = 5; s = 12; A, I) = C_4^2 C_2^1 C_1^1 = 12$.

$s = 13$, $(1,1,3,0,0)$, $FP(n = 5; s = 13; A, I) = C_4^1 C_3^3 = 4$.

$s = 14$, $(1,1,2,1,0)$, $FP(n = 5; s = 14; A, I) = C_4^1 C_3^2 C_1^1 = 12$.

$s = 15$, $(1,1,1,2,0)$, $FP(n = 5; s = 15; A, I) = C_4^1 C_3^1 C_2^2 = 12$.

$s = 15$, $(1,1,1,2,0)$, $FP(n = 5; s = 16; A, I) = C_4^1 C_3^1 C_2^1 C_1^1 = 24 = 4!$.

Method 2. Use the proposition 3 for places II, III, IV, V. We obtain the values:

$s = 1$, $(5,0,0,0,0)$, $T = 1$

$A1I = 1$	$A1II = 0$	$A1III = 0$	$A1IV = 0$	$A1V = 0$

$s = 2$, $(4,1,0,0,0)$, $T = 5$

$A2I = 4$	$A2II = 1$	$A2III = 0$	$A2IV = 0$	$A2V = 0$

$s = 3$, $(3,2,0,0,0)$, $T = 10$

$A3I = 6$	$A3II = 4$	$A3III = 0$	$A3IV = 0$	$A3V = 0$

$s = 4$, $(3,1,1,0,0)$, $T = 20$

$A4I = 12$	$A4II = 4$	$A4III = 4$	$A4IV = 0$	$A4V = 0$

$s = 5$, $(2,3,0,0,0)$, $T = 10$

$A5I = 4$	$A5II = 6$	$A5III = 0$	$A5IV = 0$	$A5V = 0$

$s = 6$, $(2,2,1,0,0)$, $T = 30$

$A6I = 12$	$A6II = 12$	$A6III = 6$	$A6IV = 0$	$A6V = 0$

$s = 7$, $(2,1,2,0,0)$, $T = 30$

$A7I = 12$	$A7II = 6$	$A7III = 12$	$A7IV = 0$	$A7V = 0$

$s = 8$, $(2,1,1,1,0)$, $T = 60$

$A8I = 24$	$A8II = 12$	$A8III = 12$	$A8IV = 12$	$A8V = 0$

$s=9,\ (1,4,0,0,0),\ T=5$

$A9I=1 \quad A9II=4 \quad A9III=0 \quad A9IV=0 \quad A9V=0$

$s=10,\ (1,3,1,0,0),\ T=20$

$A10I=1 \quad A10II=4 \quad A10III=0 \quad A10IV=0 \quad A10V=0$

$s=11,\ (1,2,2,0,0),\ T=30$

$A11I=6 \quad A11II=12 \quad A11III=12 \quad A11IV=0 \quad A11V=0$

$s=12,\ (1,2,1,1,0),\ T=60$

$A12I=12 \quad A12II=24 \quad A12III=12 \quad A12IV=12 \quad A12V=0$

$s=13,\ (1,1,3,0,0),\ T=20$

$A13I=4 \quad A13II=4 \quad A13III=12 \quad A13IV=0 \quad A13V=0$

$s=14,\ (1,1,2,1,0),\ T=60$

$A14I=12 \quad A14II=12 \quad A14III=12 \quad A14IV=12 \quad A14V=0$

$s=15,\ (1,1,1,2,0),\ T=60$

$A15I=12 \quad A15II=12 \quad A15III=12 \quad A15IV=24 \quad A15V=0$

$s=16,\ (1,1,1,1,1),\ T=120$

$A16I=24 \quad A16II=24 \quad A16III=24 \quad A16IV=24 \quad A16V=24.$

b) Compute the total number of favorable cases $FF(n; Nume, LocX)$ for any runner in final classification.

$n=5,\ FF(n=5; A, I)=1+4+6+\cdots+12+24=150;$ similar for $B, C, D, E.$

$n=5,\ FF(n=5; A, I)=1+4+6+\cdots+12+24=150;$ similar for $B, C, D, E.$

$n=5,\ FF(n=5; A, III)=4+6+\cdots+12+24=134;$ similar for $B, C, D, E.$

$n=5,\ FF(n=5; A, IV)=12+12+\cdots+12+24=84;$ similar for $B, C, D, E.$

$n=5,\ FF(n=5; A, V)=24;$ similar for $B, C, D, E.$

c) Proof of correctitude.

$150+149+130+84+24=541=S(n=5).$ Correctly.

Wee see the abstract of all results in Table1, for $n=1,2,3,4,5.$

Centralization table 1 for $n = 1, 2, 3, 4, 5.$

	n=1	*n*=2		*n*=3			*n*=4				*n*=5					
	FP	FP		FP			FP				FP					
	I	I	II	I	II	III	I	II	III	IV	I	II	III	IV	V	
s =1	1	1	0	1	0	0	1	0	0	0	1	0	0	0	0	1
s=2		1	1	2	1	0	3	1	0	0	4	1	0	0	0	2
s=3				1	2	0	3	3	0	0	6	4	0	0	0	3
s=4				2	2	2	6	3	3	0	12	4	4	0	0	4
s=5							1	3	0	0	4	6	0	0	0	5
s=6							3	6	3	0	12	12	6	0	0	6
s=7							3	3	6	0	12	6	12	0	0	7
s=8							6	6	6	6	24	12	12	12	0	8
s=9											1	4	0	0	0	9
s=10											4	12	4	0	0	10
s=11											6	12	12	0	0	11
s=12											12	24	12	12	0	12
s=13											4	4	12	0	0	13
s=14											12	12	24	12	0	14
s=15											12	12	12	24	0	15
s=16											24	24	24	24	24	16
FF	1	2	1	6	5	2	26	25	18	6	150	149	134	84	24	
S(*n*)	1	–	3	–	–	13	–	–		75					541	

Example 10. Let $n = 5$ be the number of runners $A, B, C, D, E.$

a) Attach the random variable $X5$.

b) Compute the mean value and variance.

Solution. a) We use the values from centralization table 1

$N(5) = 16,\ S(n) = S(5) = 541$

$FFI = 150,\ FFII = 149,\ FFIII = 134,\ FFIV = 84,\ FFV = 24$

$$X5 = \begin{pmatrix} \frac{100}{1} & \frac{100}{2} & \frac{100}{3} & \frac{100}{4} & \frac{100}{5} \\ \frac{FFI}{S(n)} & \frac{FFII}{S(n)} & \frac{FFIII}{S(n)} & \frac{FFIV}{S(n)} & \frac{FFV}{S(n)} \end{pmatrix}$$

$$X5 = \begin{pmatrix} 100 & 50 & 33{,}3334 & 25 & 20 \\ \frac{150}{541} = 0{,}2773 & \frac{149}{541} = 0{,}2754 & \frac{134}{541} = 0{,}2477 & \frac{84}{541} = 0{,}1553 & \frac{24}{541} = 0{,}0443 \end{pmatrix}$$

b) Compute the mean value and variance

$M(X5) = 100 \cdot 0{,}2773 + 50 \cdot 0{,}2754 + 33{,}3334 \cdot 0{,}2477 + 25 \cdot 0{,}1553 + 20 \cdot 0{,}0443$

$M(X5) = 54{,}525$ points.

$$(X5)^2 = \begin{pmatrix} 10000 & 2500 & 1111{,}1155 & 625 & 400 \\ 0{,}2773 & 0{,}2754 & 0{,}2477 & 0{,}1553 & 0{,}0443 \end{pmatrix}$$

$$D^2(X) = M(X)^2 - [M(X)]^2 = 3851{,}5058 - 2972{,}9756 = 878{,}5302$$

$D^2(X5) = 878{,}5302, D(X5) = 29{,}64$ points.

Final remark. For $n = 3,4,5,6$ we have the following statistical results.

1) $M(X3) = 70{,}5067$, $M(X4) = 61{,}67$, $M(X5) = 54{,}525$, $M(X6) = 49{,}2354$.

If number n is increasing, then the mean value of score is decreasing (↓).

2) $D(X3) = 27{,}8604$, $D(X4) = 28{,}5303$, $D(X5) = 29{,}64$, $D(X6) = 29{,}601$.

If number n is increasing, then the standard deviation of score is increasing (↑).

REFERENCES

[1] Popoviciu Nicolae, *Special chapters of probability and statistics*, Victor Publishing, Hyperion University of Bucharest, ISBN 978-973-1815-98-5, 2014, p. 338.