INVESTIGATION OF NONLINEAR CORRELATION BETWEEN PRIME INDIAN AND AMERICAN STOCK EXCHANGE INDICES USING EMPIRICAL MODE DECOMPOSITION

Swetadri SAMADDER*, Koushik GHOSH** and Tapasendra BASU***

Abstract. *Since last few decades there have been a number of observations indicating the possibility that the behaviour of American stock exchange may have a significant influence on the Indian stock market behaviour. The present work is an effort in this direction and the purpose of the present work is to investigate a nonlinear correlation between Indian and USA stock markets by means of Empirical Mode Decomposition (EMD). For the present analysis we have taken into consideration the prime Indian stock market indices viz. Sensex and Nifty and the prime US stock market index DOW JONES. The aim of the present study is to analyze the nature of the probability distribution of the indices, their phase distributions, amplitude distributions and also a comparison between them. We first examined the probability distribution of the normalized logarithmic return time series generated by the respective data. We also examined shape of the probability distribution and compared their similarity/dissimilarity. Next we applied Hilbert-Huang transform on the logarithmic return time series to analyse their periodicity and properties. Finite numbers of IMF are obtained by EMD Method. We also calculated probability distribution of instantaneous amplitude and phase of these IMF and compared the same for the present three time series. Our result shows striking similarity of the behaviour of Dow Jones with both Sensex and Nifty.*

Keywords: *Probability distribution; Empirical Mode Decomposition; Hilbert-Huang Transform; Intrinsic Mode Function, Amplitude, Phase.*

1. Introduction

Stock market data is often non-stationary, non-linear and non-homogeneous in nature. For this type of dynamical systems, most of the

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research about their experimental data actually focuses on the explanation of the observed phenomenon and thus the method of analyzing the data plays an important role in the way to the experimental outcome. Therefore, how to extract the essential components from the experimental data is an important area of research in the field of financial data analysis.

Bachelier [1] was first to propose theory of random walk to model the financial time series. He showed that empirical evidence seemed to confirm random walk hypothesis: today's change in price is not effected by yesterday's behaviour and it totally depends on the situation of today's market only. It means there are no memory effects and returns in each day are independent and random. Since then many new researchers worked on analysing the stock market data and opposing the view of Bachelier [1] there have been some communications claiming some kind of memory flow in the financial time series with some hints of quasi-periodicity [2-5]. As stock market is non-homogeneous and complex, the nature of the underlying periodicity is rather difficult to capture exhaustively. There are some studies to identify periodicity of stock market indices, but those works have not been able to analyse exact nature of the periodicity. Zhou [6] indicated that the heavy tail of financial time series is mostly caused by the heteroscedasticity of the time series.

One of the major drawbacks of all these studies is limited scope provided by the statistics of conventional derivatives from returns. But recently the application of analytical physics in economics generates a new branch of exploration called "Econophysics". Econophysics is an interdisciplinary research field, applying theories and methods originally developed by physicists in order to solve problems in economics, usually those including uncertainty or stochastic processes and nonlinear dynamics [7-9].

Several important works have been done in this new field of research [10-17]. A new technique has been introduced by Huang *et al.* [18] to analyse periodicity and properties of a time series. This method is known as Empirical Mode Decomposition (EMD). Different from conventional filters, EMD is a method good at extracting the physical tempo or trend from experiment data. This advantage enables researchers to provide a physical explanation about time series. This method is based on Hilbert view. The key part of this technique is that it generates finite and often a small numbers of intrinsic mode functions (IMF) that assume well-behaved Hilbert- Huang Transform. This technique was applied in many works and experienced noteworthy successes [19-24].

In the present work, we have tried to investigate a nonlinear correlation between the behaviour of Indian and USA stock markets by using EMD method. The reason behind this investigation is that there have been several observations since last few decades which indicate a possible influence of US stock market on the behaviour of Indian stock market. For the present analysis we have taken into consideration the prime Indian stock exchange indices viz. Sensex [25] and Nifty [26] which are observed to be persistent in nature [27] and very well known American stock exchange index Dow Jones [25]. The aim under study is to obtain the nature of the probability distribution of the indices, their phase distributions, amplitude distributions and also a comparison between the same.

We first examine probability distribution of the normalized logarithmic return time series generated by the respective data. We also examine shape of the probability distribution and compare their similarity/dissimilarity. We then apply Hilbert-Huang transform on the logarithmic return time series to analyse their periodicity and properties. Finite numbers of Intrinsic Mode Function are obtained by Empirical Mode Decomposition Method. We also calculate probability distribution of instantaneous amplitude and phase of the IMF's and compare the same for three time series.

2. Theory

1.1. Distribution of Time Series Return

Let the closing price of the index be $I(t)$. The time series of logarithmic returns of $I(t)$ over a time scale of τ is defined as

$$R_\tau(t) = \text{In}\,\frac{I(t)}{I(t-\tau)} \tag{1}$$

where τ is the primary unit, in this case 1 day. As τ is a parameter used to sample time series of returns, we can take different τ for $R_\tau(t)$ to analyse the behaviour of the return time series with interday frequencies. We define the normalized logarithmic returns as

$$R_\tau(t) = \text{I}\,\frac{R_\tau(t) - \overline{R}_\tau(t)}{\sqrt{\overline{R}_\tau^2(t) - (\overline{R}_\tau(t))^2}} \tag{2}$$

where the expectation values denoted by $\overline{R}_\tau(t)$ are taken over the entire time period under consideration.

We further define the probability density function P as the normalised distribution of a measure ρ, which satisfies

$$\int_{-\infty}^{+\infty} P(\rho)d\rho = 1 \tag{3}$$

where the measure ρ can be $R_\tau(t)$, $r_\tau(t)$, $R_\tau(t)$, phase or amplitude defined in the later discussions. P can be calculated from histogram of the data using kernel density estimation.

1.2. Empirical Mode Decomposition (EMD)

1.2.1. Background

For a given signal $s(t)$, we search for a decomposition into simple signals (modes)

$$s(t) = \sum_{j=1}^{M} a_j(t) \cos \varphi_j(t) \tag{4}$$

where $a_j(t)$ is the amplitude and $\varphi_j^{\,t}$ is the phase of the *jth* component. Each of the components should have a physical and mathematical meaning. Let us consider $s(t)$ as a mono-component signal. Then $s(t)$ can be represented as

$$s(t) = a(t) \cos \theta(t) \tag{5}$$

that is both physically and mathematically meaningful. There are infinitely many ways to construct such representations, but there is advantage to write the signal in complex form

$$S(t) = s(t) + is_1(t) = A(t)e^{i\varphi(t)} \tag{6}$$

and to take the actual signal as the real part of the complex signal. The imaginary part $s_1(t)$ has to be chosen with a physical and mathematical meaning. If it is done, we can define the amplitude and phase by

$$A(t) = \sqrt{s^2 + s_1^{\,2}} \tag{7}$$

and

$$\varphi(t) = \tan^{-1}\left(\frac{s}{s_1}\right). \tag{8}$$

1.2.2. Basic Idea of EMD

The basic idea of the empirical mode decomposition is to consider signals at the level of their local oscillations. Looking at the generation of a signal $x(t)$ between two consecutive local extrema (say, two minima occurring at times $t-$ and $t+$), we can define a high frequency part $\{d(t), t- \leq t \leq t+\}$, called detail locally. $d(t)$ corresponds to the oscillation which terminates at the two minima and passes through the maximum which must exists in between them. We also identify the corresponding low-frequency part $m(t)$ locally, or local trend. Thus we have $x(t) = m(t) + d(t)$ for $t- \leq t \leq t+$. Assuming that this is done in some proper way for all the oscillations composing the entire signal, we get an Intrinsic Mode Function (IMF) as well as a residual consisting of all local trends. The same procedure can then be applied to this residual, considering it as a new signal to decompose, and successive constitutive components of a signal can therefore be iteratively extracted such that each component is locally (i.e., at the scale of one single oscillation) in the highest frequency band.

Given a signal $x(t)$, the effective algorithm of EMD can be summarised as follows:

1. Identify all extrema of $x(t)$;

2. Interpolate between minima (respectively, maxima), ending up with some envelope *emin*(t) (respectively, *emax*(t));

3. Compute the mean m(t) = (emin(t) + emax(t))/2;

4. Extract the detail $d(t) = x(t) - m(t)$;

5. Iterate on the residual $m(t)$.

In practice, the above procedure has to be refined by a sifting process which consists of iterating steps 1 to 4 upon $d(t)$, until $d(t)$ can be considered as zero-mean according to some stopping criterion. Once this is achieved, $d(t)$ is considered as the effective IMF, the corresponding residual is computed and step 5 applies.

By construction, the number of extrema is decreased when going from one residual to the next and by this way it is guaranties that the complete decomposition of the signal is achieved in a finite number of steps. Modes and residuals can spontaneously be given a "spectral" interpretation. But in the general case, their high vs. low frequency discrimination applies only locally and corresponds by no way to a pre-determined sub-band filtering (as, for e.g., in a wavelet transform). Selection of modes rather corresponds to an automatic and signal-dependent time-variant filtering.

1.2.3. EMD Analysis of the Time Series

The algorithm for generating IMF consists of two steps:

Step 1: The local extreme value of $R_\tau(t)$ are identified and are connected by cubic spline line. The local maximum vales form the upper envelope $U(t)$, and the local minimum values form the lower envelope $L(t)$. These two envelopes will encompass all data points. The average of upper and lower envelopes, $m_1(t)$ is

$$m_1(t) = \frac{U(t) + L(t)}{2}. \tag{9}$$

Then $m_1(t)$ is subtracted from $R_\tau(t)$ to get a component $h_1(t)$

$$h_1(t) = R_\tau(t) - m_1(t) \tag{10}$$

$h_1(t)$ is an IMF if it satisfies the following conditions:

$h_1(t)$ is free from riding waves.

$h_1(t)$ displays symmetry of $L(t)$ and $U(t)$ w.r.t zero.

The numbers of zero crossing and extremes are the same or only differ by 1.

If $h_1(t)$ is not an IMF, we continue to perform the previous steps until the extracted $h_1(t)$ is an IMF.

In this process $h_1(t)$ is treated as the original data and the step

$$h_{11}(t) = h_1(t) - m_{11}(t)$$ (11) is repeated.

If the function $h_{11}(t)$ still can't reach the requirement of an IMF, k times of the above step are repeated until the obtained function reaches an acceptable threshold value to be an IMF. The result is

$$h_{1k}(t) = h_{1(k-1)}(t) - m_{1k}(t). \tag{12}$$

$h_{1k}(t)$ is the first IMF and is designated as $h_{1k}(t) = C_1(t)$. $C_1(t)$ contains highest oscillatory frequency found in $R_\tau(t)$.

Step 2: The first IMF $C_1(t)$ is subtracted from the original data $R_\tau(t)$ to obtain r_1:

$$r_1(t) = R_\tau(t) - C_1(t) \tag{13}$$

is a residue.Then $r_1(t)$ is treated as the original data and step 1 is repeated. Following step 1 and step 2, we can find more IMF C_i until the last one. The last residue is a constant value or a monotonical trend. Then

$$R_\tau(t) = \sum_{i=1}^{n} C_i(t) + r_n \tag{14}$$

and

$$r_{i-1}(t) - C_i(t) = r_i(t) \tag{15}$$

are obtained.

1.2.4. Amplitude and Instantaneous Phase

After all IMF's are obtained by EMD method, we can calculate amplitudes and instantaneous phases of IMF's by applying Hilbert transform to each IMF component. For the rth component $C_r(t)$, the computation of Hilbert Transform includes the conjugation of $C_r(t)$:

$$y_r(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{C_r(t')}{t - t'} dt' \tag{16}$$

where P is the Cauchy principal value. From this definition, two function $C_r(t)$ and $y_r(t)$ will make a complex conjugate pair and define an analytical signal $Z_r(t)$:

$$Z_r(t) = C_r(t) + iy_r(t) \equiv A_r(t) e^{i\phi(t)} \tag{17}$$

where the amplitude $A_r(t)$ and instantaneous phase $\varnothing_r(t)$ are defined as

$$A_r(t) = [c_r^2(t) + y_r^2(t)]^{\frac{1}{2}} \tag{18}$$

and

$$\phi_r(t) = \tan^{-1}\left(\frac{y_r(t)}{C_r(t)}\right). \tag{19}$$

As for statistics phase, probability density function is further defined. The probability density P is defined as the normalized probability distribution of observed quantity $\rho,$ which satisfy

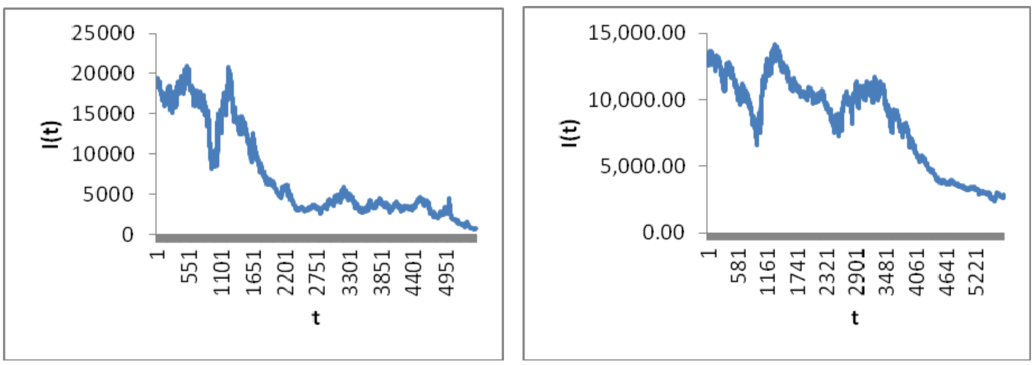
$$\int_{-\infty}^{+\infty} P(\rho)\, d\rho = 1 \tag{20}$$

where ρ could be phase or amplitude. P can be calculated from histogram of the data using kernel density estimation.

2. Results

For the present analysis we have considered the daily close data of two prime Indian stock indices: Sensex & Nifty and one prime American stock index: Dow Jones. Our aim is to compare Dow Jones index with Sensex and Nifty separately to check if there is any similarity between them. For this purpose, firstly, we have taken the interval for comparative computation between Sensex and Dow Jones in the range from 1st January, 1990 to 31st December, 2012 [25, 26]. Secondly, the interval for comparative computation between Nifty and Dow Jones is taken from 3rd July, 1990 to 31st December, 2012 [25]. These time intervals are chosen as per the common availability of the data for two competitive indices. For the first set of data, we have 5492 and 5797 data point for Sensex and Dow Jones respectively and for the second set of data; we have 5287 and 5670 data point for Nifty and Dow Jones respectively.

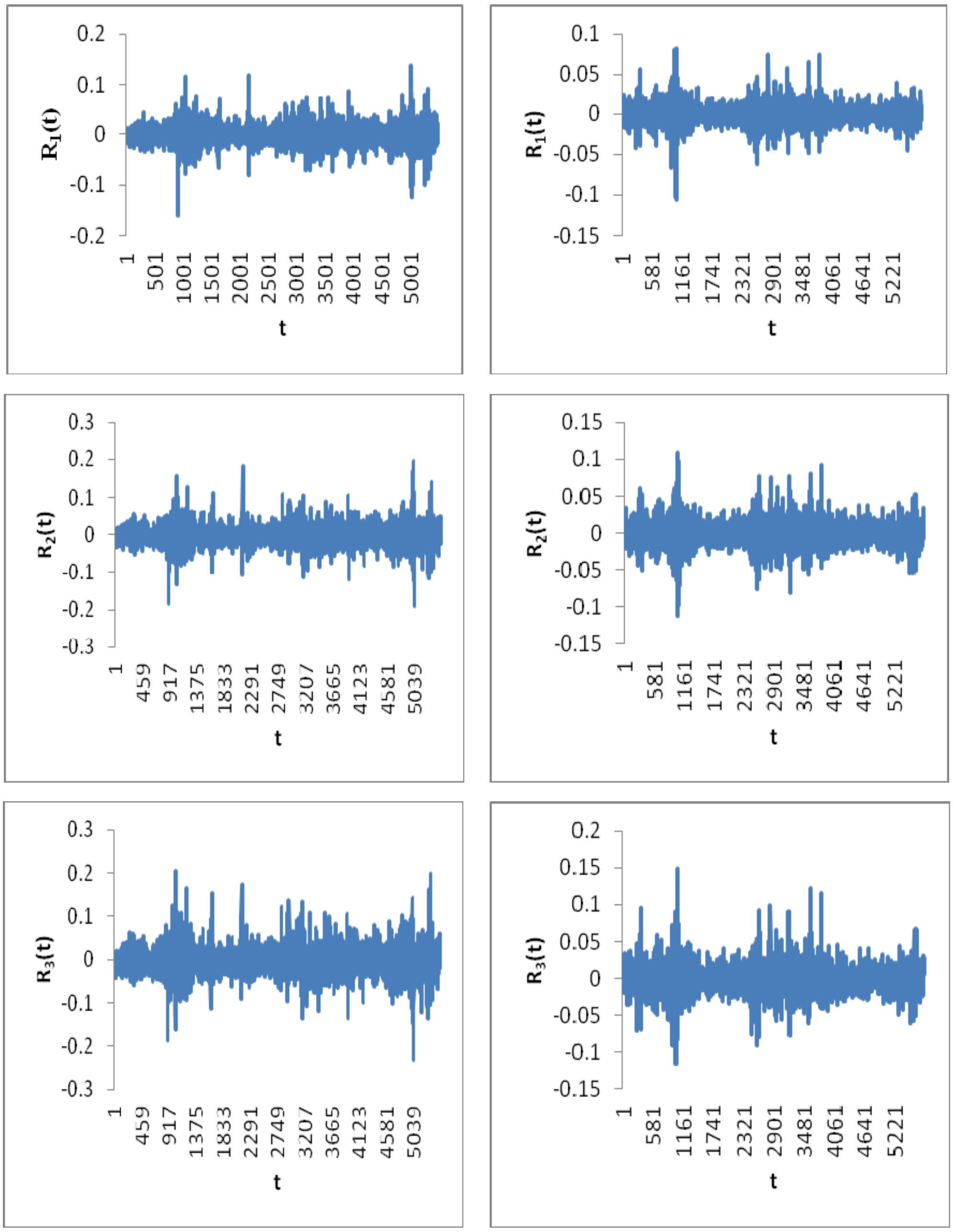
Fig. 1a and b shows closing price graph of Sensex and Dow Jones respectively.

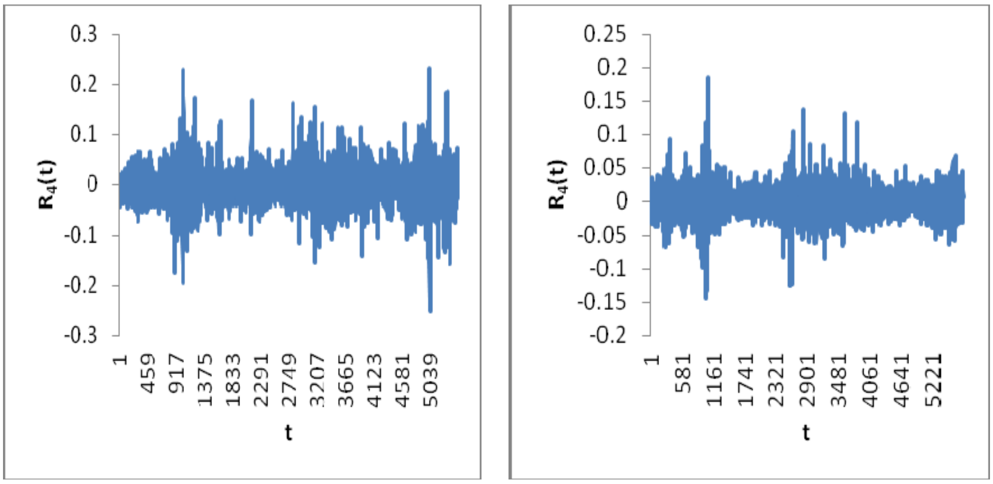


(a) **(b)**

Figure 1. Daily Closing Value plots for **(a)** Sensex; **(b)** Dow Jones.

We have first calculated $R_\tau(t)$ for $\tau = 1-4$ days for Sensex and Dow Jones and results obtained are shown in Fig. 2a and b.

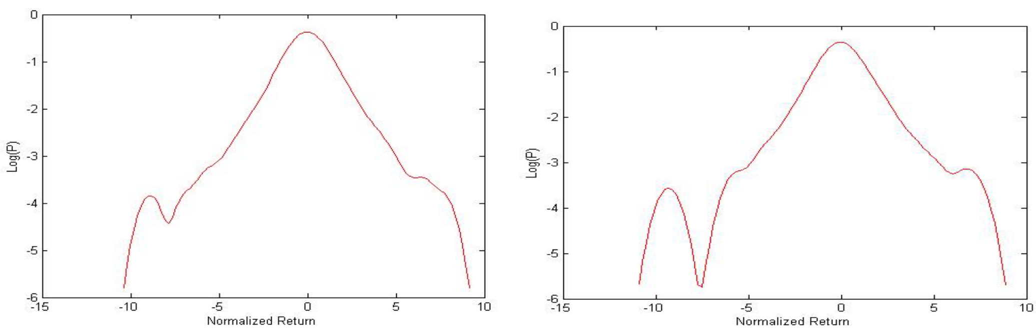




(a) (b)

Figure 2. Plots for time series data of log returns $R_\tau(t)$ of daily closing value of **(a)** Sensex and **(b)** Dow Jones, sampled by 1, 2, 3 and 4 days.

Next, we have estimated probability density function of normalized returns of the data by kernel density estimation and the distribution is given in Fig. 3a and b for Sensex and Dow Jones respectively.

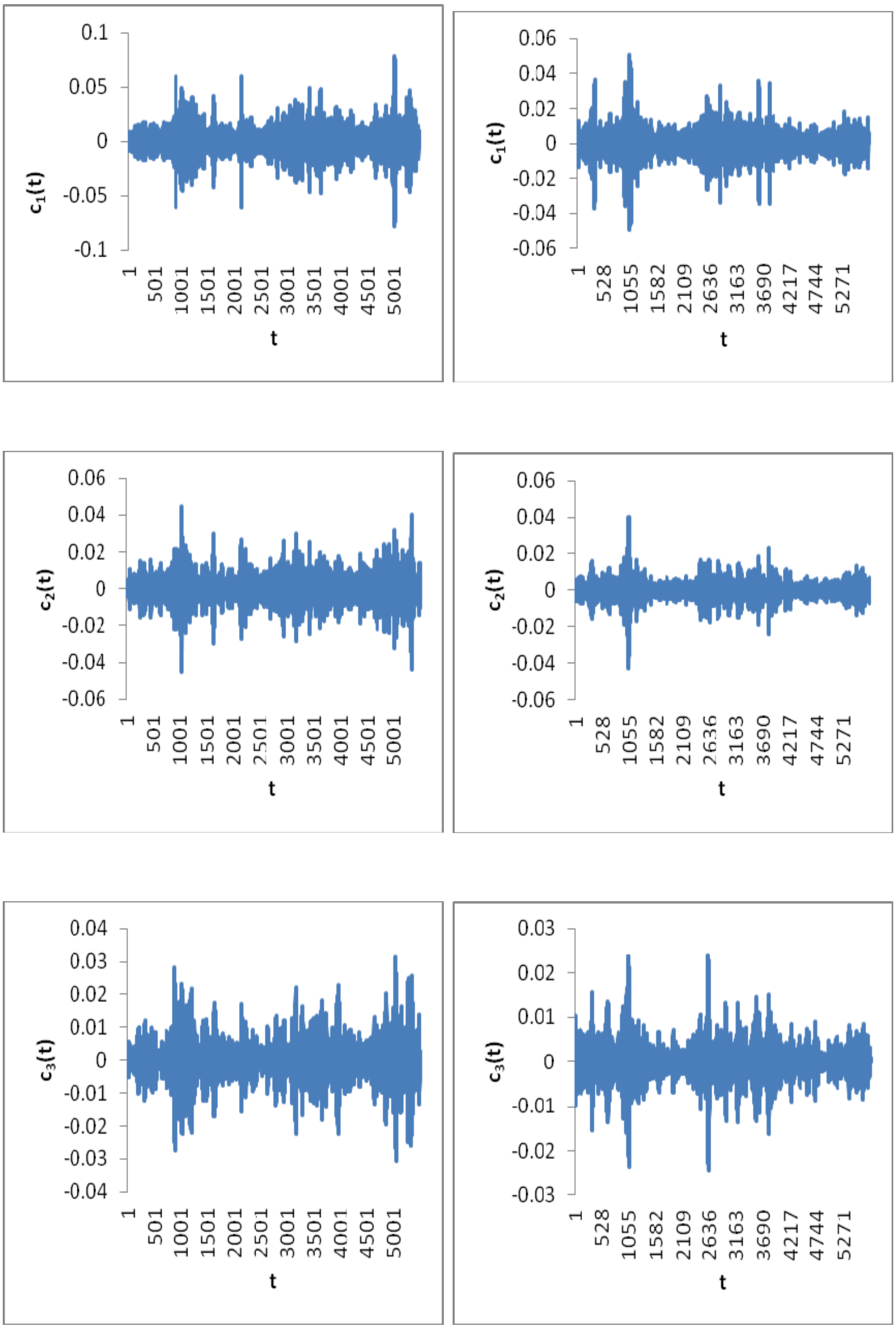


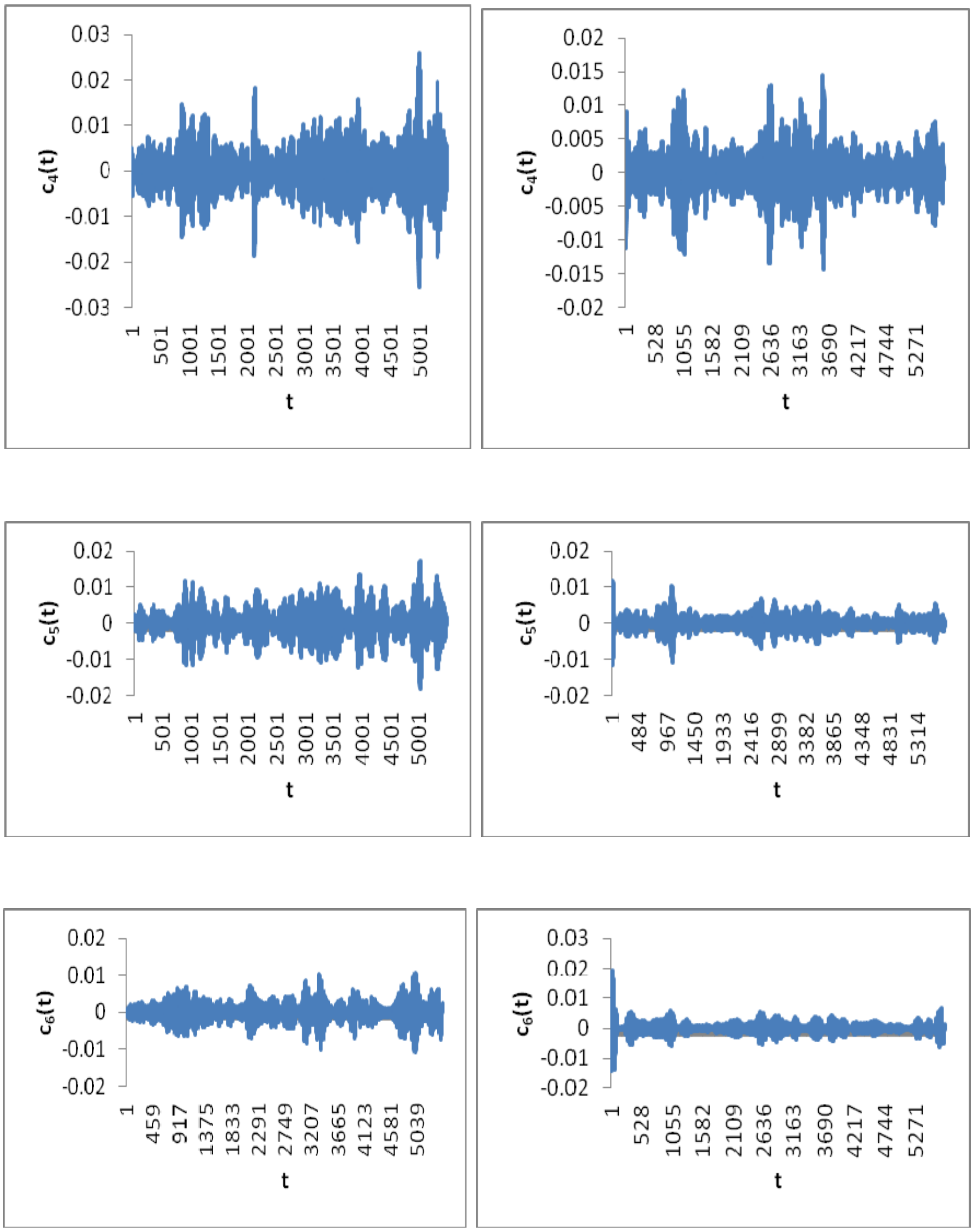
(a) (b)

Figure 3. Plots for probability distribution of normalized returns of **(a)** Sensex and **(b)** Dow Jones.

It is noticeable that values of normalized return of Sensex are ranging from – 8.90 to 7.66 and those of Dow Jones are ranging from – 9.40 to 7.38.

Next, we have performed EMD to decompose logarithmic return $R(t)$ with time sampling interval 1 day. We have obtained 17 IMFs for Sensex and 18 IMFs for Dow Jones. In Fig. 4a and b only 6 IMFs are shown for Sensex and Dow Jones respectively.

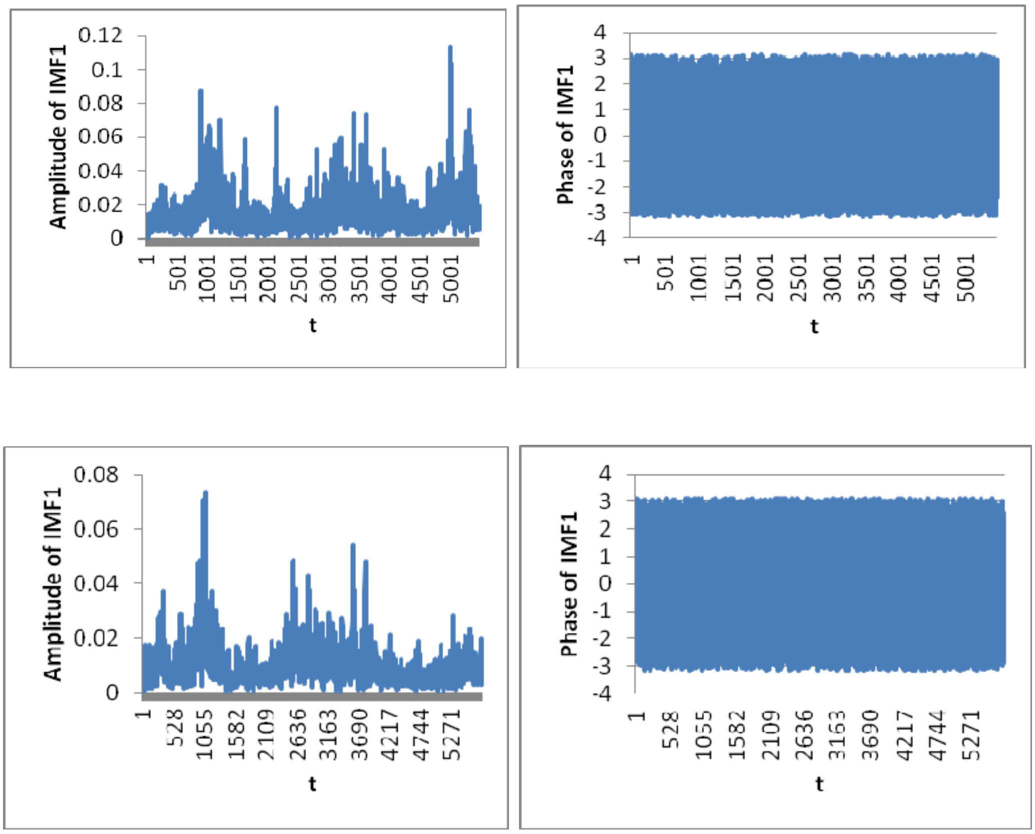




(a) (b)

Figure 4. Plots for first 6 IMFs for **(a)** Sensex and **(b)** Dow Jones.

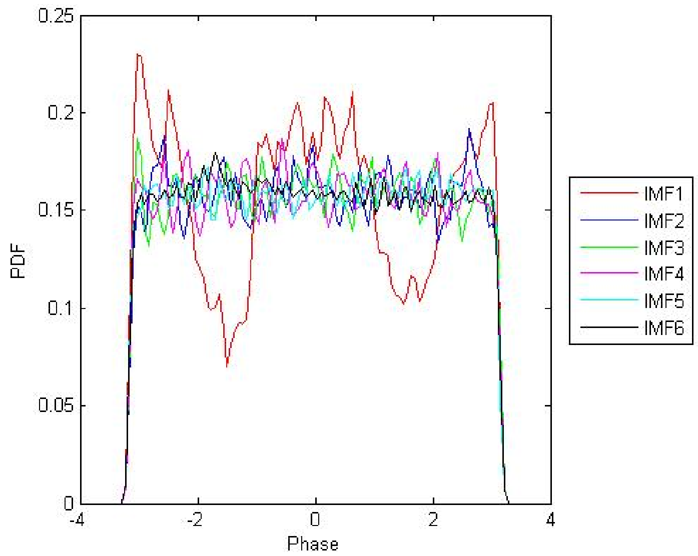
Then we have worked on the amplitudes and the phases of the IMFs for Sensex and Dow Jones by Hilbert transform. The graphs of the amplitude and phases for the first IMF are shown in Figure 5.



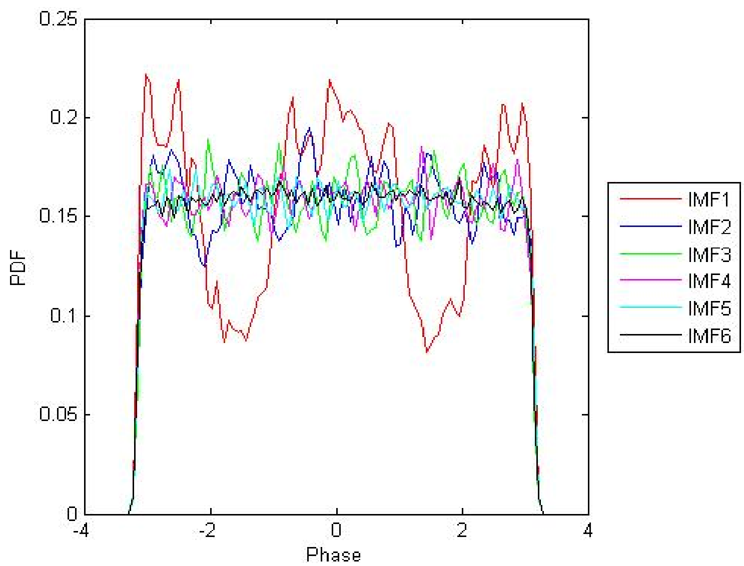
(a) (b)

Figure 5. Plots for Amplitude and Phase of the first IMFs for **(a)** Sensex and **(b)** Dow Jones.

Next we have calculated the phase distribution of the 1^{st} 6 IMFs of the two time series and result is given in Figure 6.



(a)

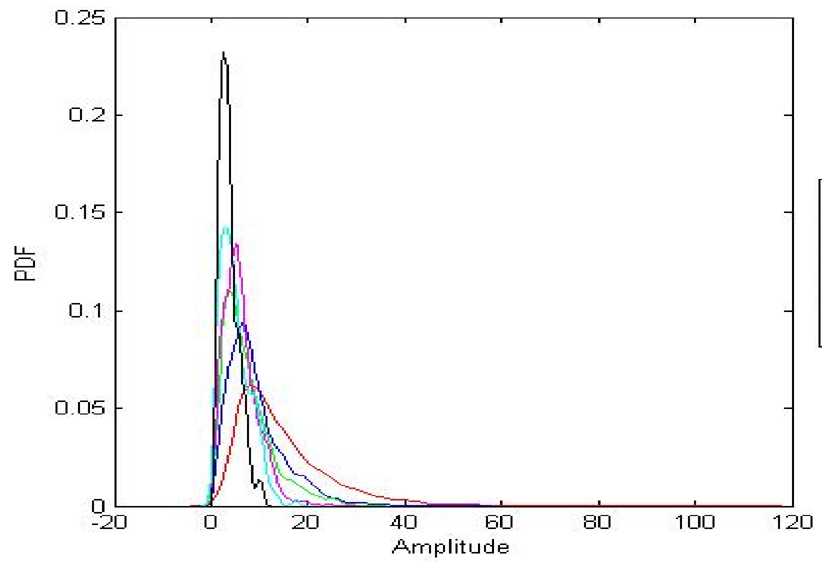


(b)

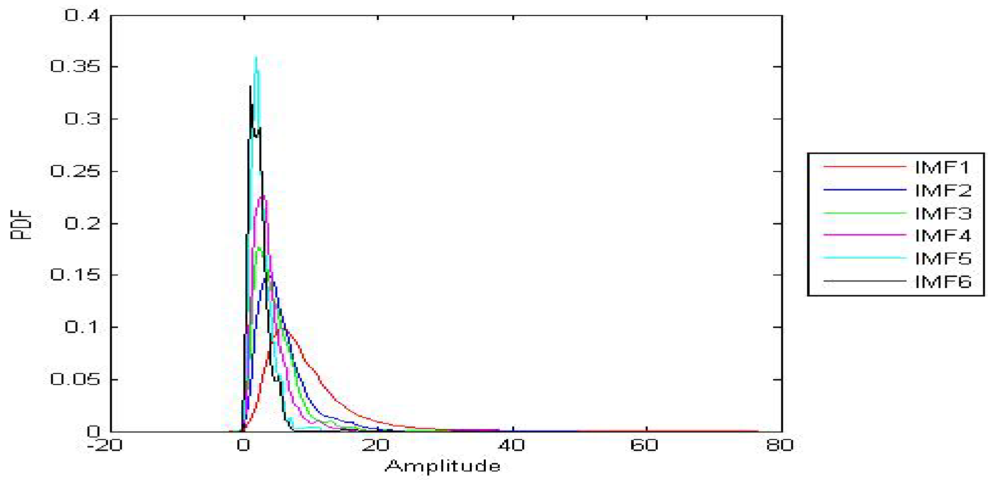
Figure 6. Plots for probability distributions of phases of 1st 6 IMFs of (**a)** Sensex and **(b)** Dow Jones.

We find that values of the phase lie between – 3.14 to 3.14 for both Sensex and Dow Jones.

After that, we have looked at the amplitude distribution of the 1st 6 IMFs of the two time series and result is given in Figure 7. Magnitude of the amplitude is much smaller than the phase. So, we have adjusted the amplitude by multiply it by 1000 to fit it in the same scale for phase.



(a)

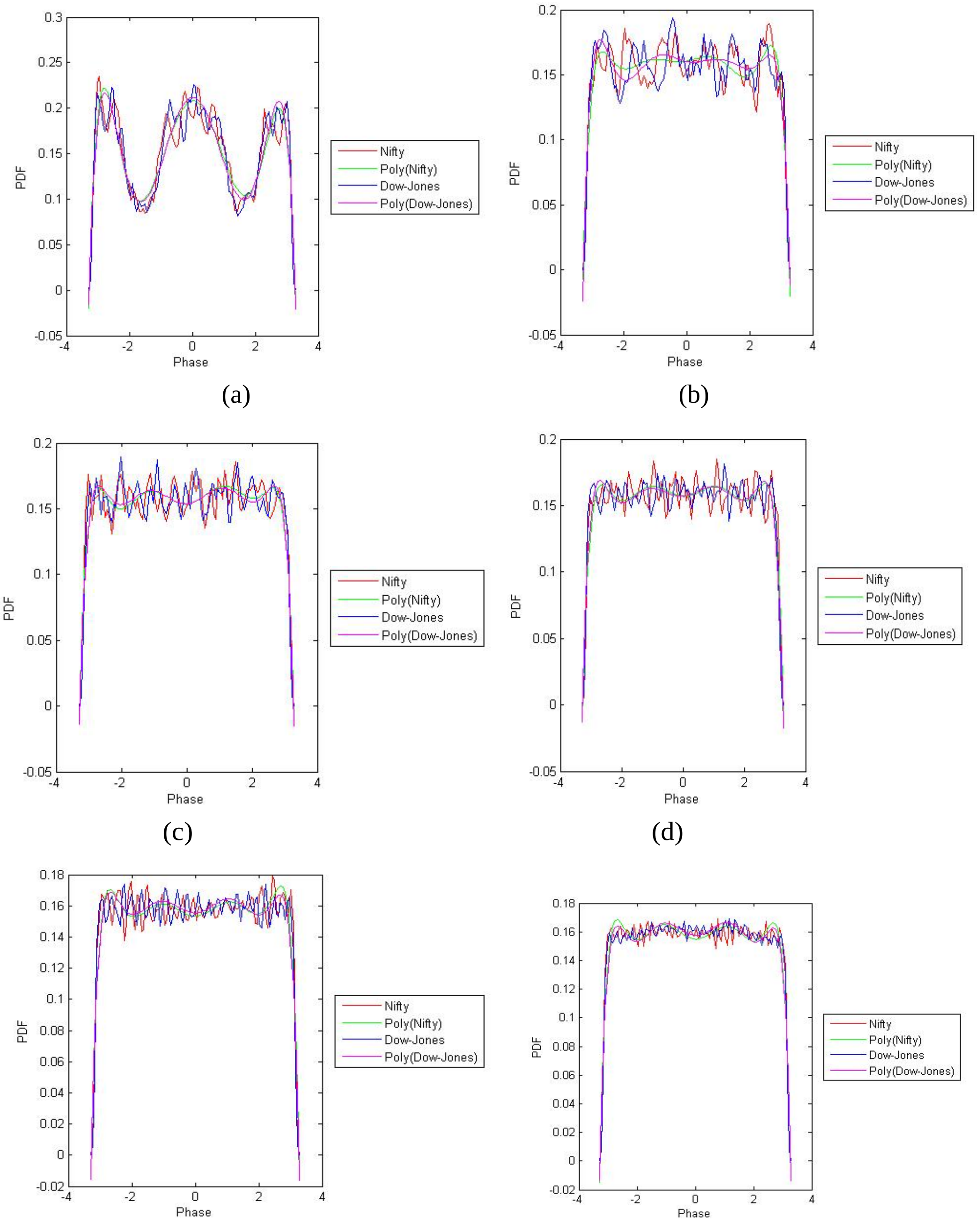


(b)

Figure 7. Plots for probability distributions of amplitudes of 1st 6 IMFs of **(a)** Sensex and **(b)** Dow Jones.

It is clear from Fig. 7a and b that range of values of amplitude is much higher in Sensex than Dow Jones.

Next we have done a comparative study of the respective phase distributions of Sensex and Dow Jones for the first 6 IMFs. We have also plotted corresponding best fitting polynomial regression curves of order 9. Figure 8 illustrates the results.



(a) (b) (c) (d) (e) (f)

Figure 8. Plots for probability distributions of phases of 1st 6 IMFs of Sensex and Dow Jones depicting polynomial regression curves of order 9.

As the amplitude value range differs in Sensex and Dow Jones, we have normalized same and compare them with best fitting polynomial curve of order 9.

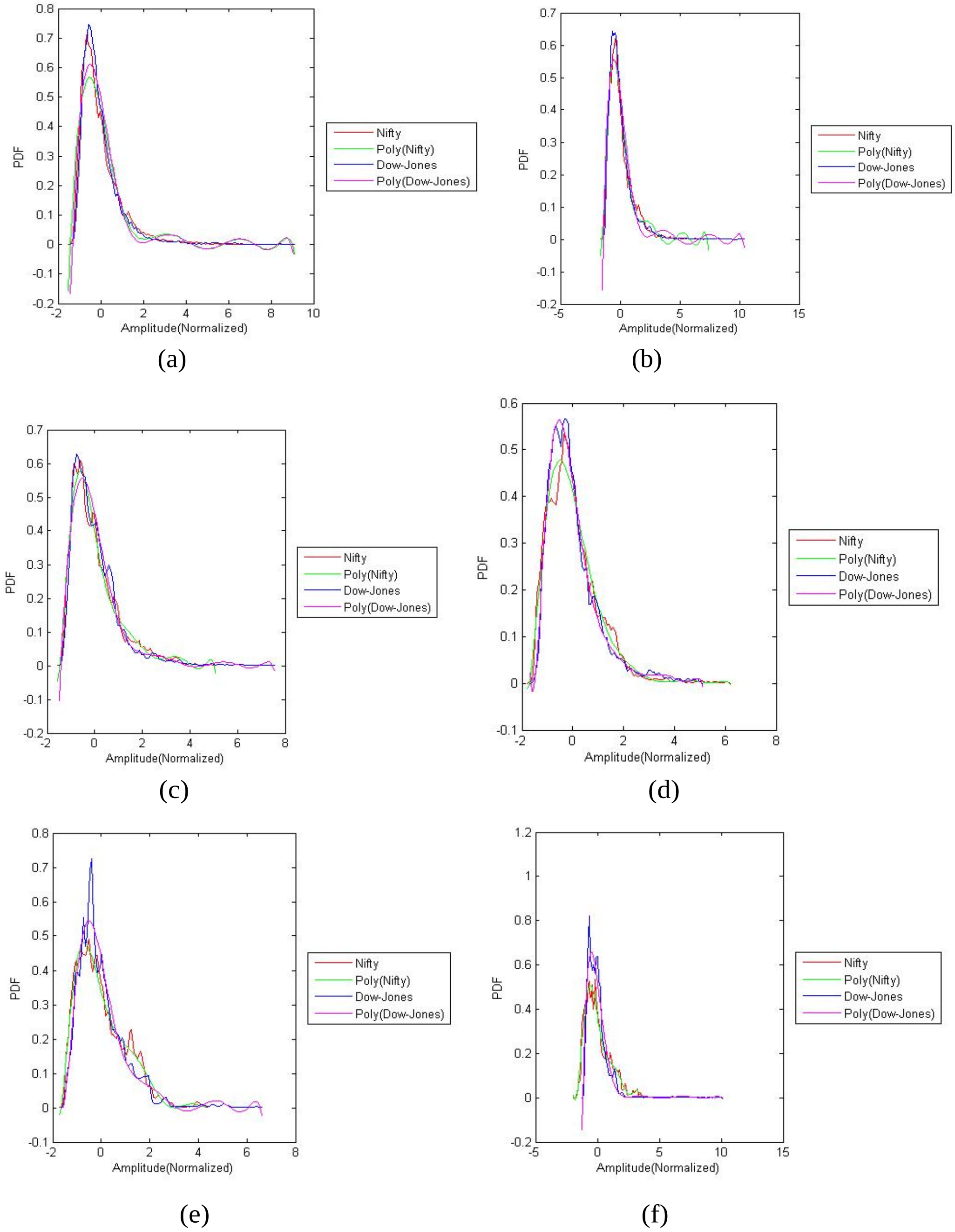


Figure 9. Plots for probability distributions of amplitudes (normalized) of 1st 6 IMFs of Sensex and Dow Jones depicting polynomial regression curves of order 9.

Now we have concentrated on the 2nd set of data which consists of daily closing price of Nifty and Dow Jones starting from 3rd July, 1990 to 31st December, 2012. We have performed same calculations mentioned above for this pair of data.

Figure 10a and b shows closing price graph of Nifty and Dow Jones respectively.

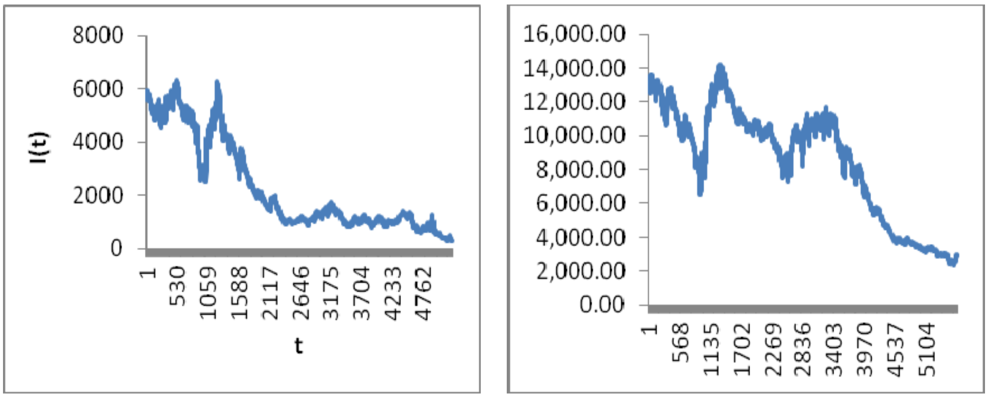
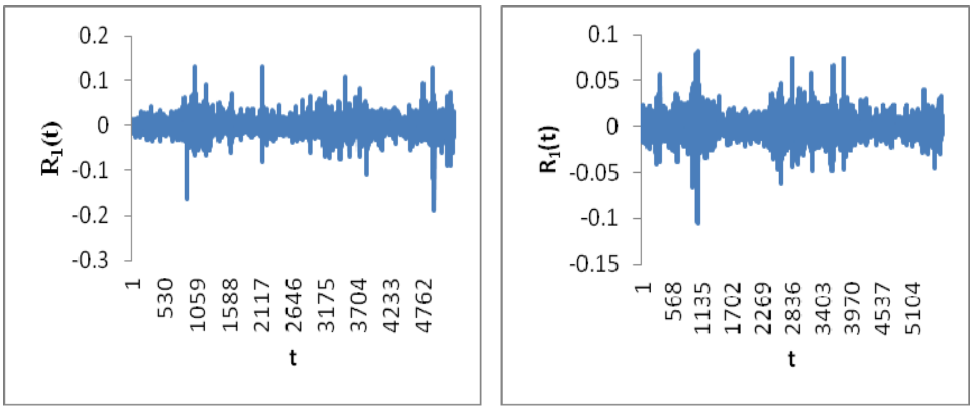


Figure 10. Daily Closing Value plots for **(a)** Nifty; **(b)** Dow Jones.

We have found $R_\tau(t)$ for $\tau = 1-4$ days for Nifty and Dow Jones and results obtained are shown in Figure 11a and b.



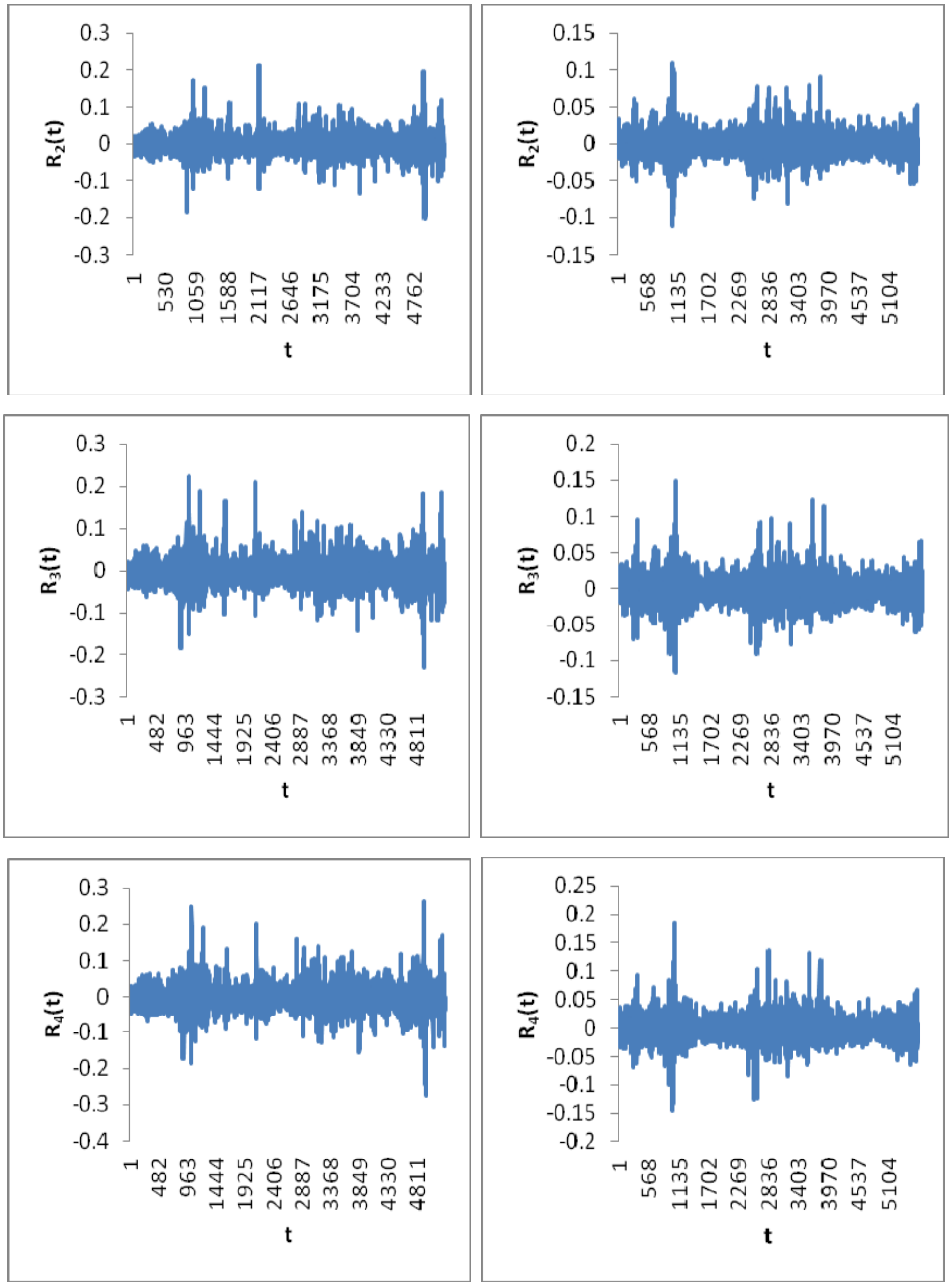
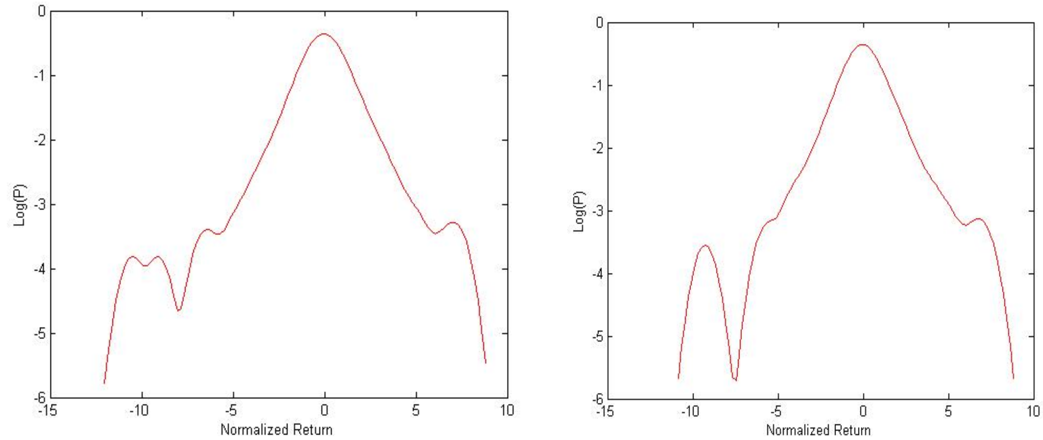


Figure 11. Plots for time series data of log returns $R_\tau(t)$ of daily closing value of **(a)** Nifty and **(b)** Dow Jones, sampled by 1, 2, 3 and 4 days.

Probability density function of the normalized return of Nifty and Dow Jones is shown is Fig. 12a and b respectively.

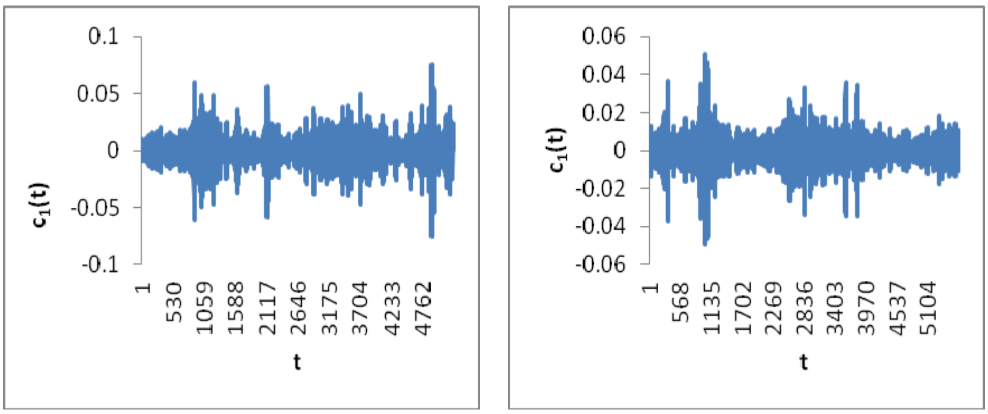


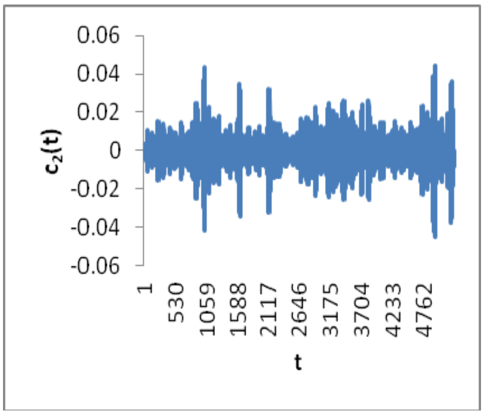
(a) (b)

Figure 12. Plots for probability distribution of normalized returns of **(a)** Nifty and **(b)** Dow Jones.

Here we observe that values of normalized return of Nifty are ranging from – 10.52 to 7.32 and those of Dow Jones are ranging from – 9.35 to 7.34.

Next, we have worked on EMD on $R(t)$ with time sampling interval 1 day and have obtained 18 IMFs for both Nifty and Dow Jones. In Figure 13a and b only 6 IMFs are shown for Nifty and Dow Jones respectively.

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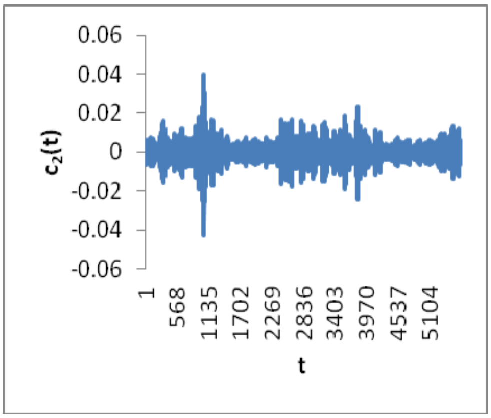
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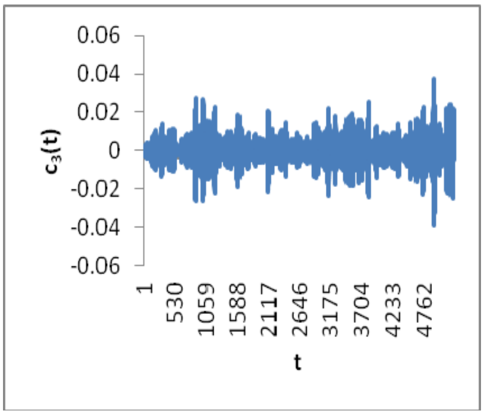
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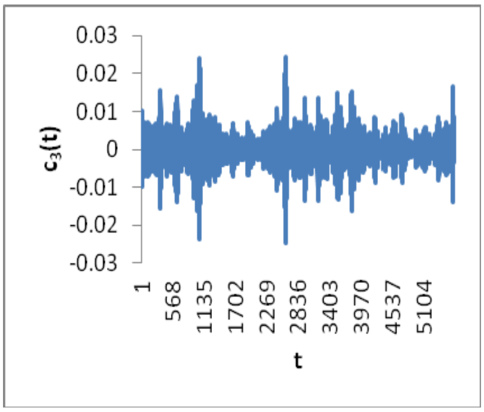
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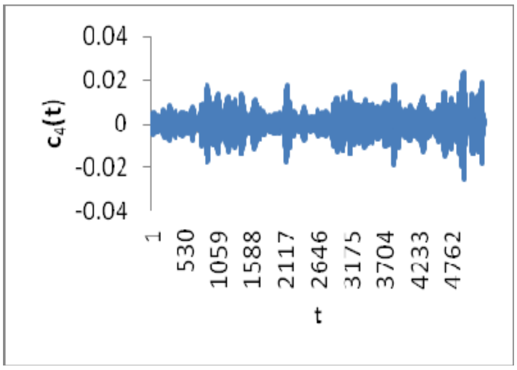
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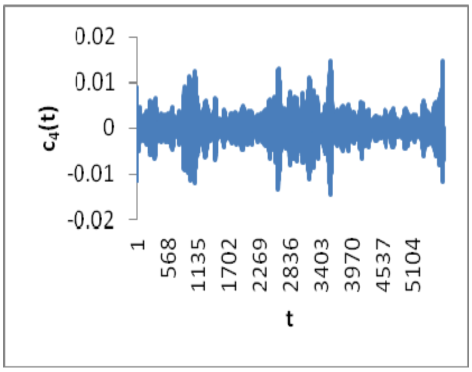
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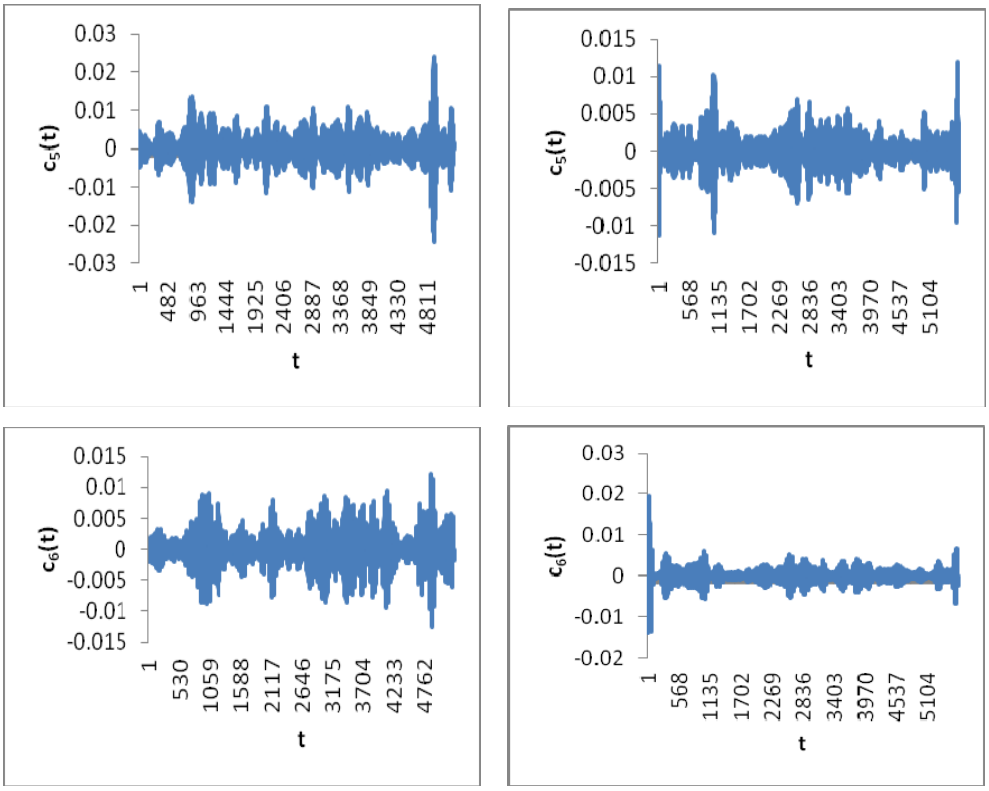
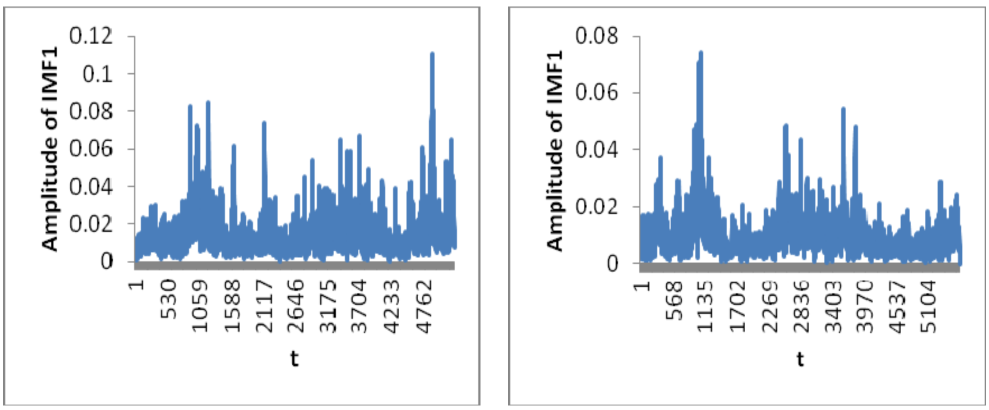
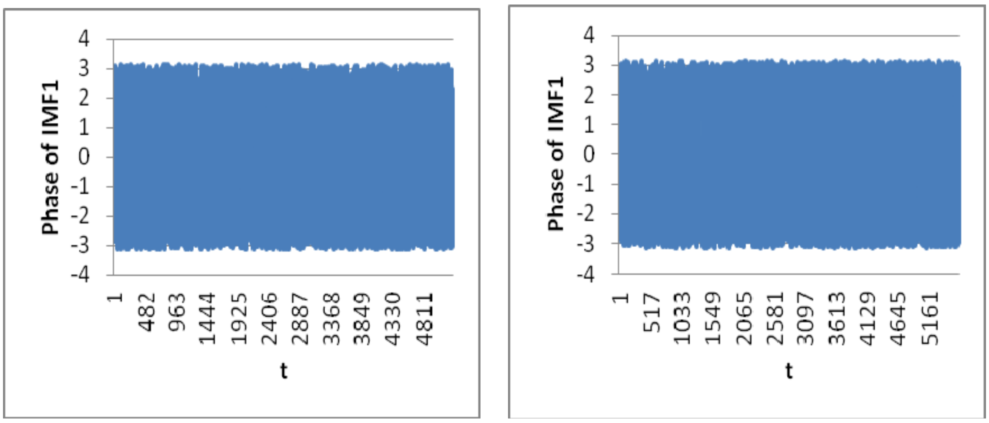


(a) (b)

Figure 13. Plots for first 6 IMFs for **(a)** Nifty and **(b)** Dow Jones.

Then we have calculated amplitudes and phases of the IMFs for Nifty and Dow Jones by Hilbert transform. The graphs of the amplitudes and phases for the first IMF are shown in Fig 14. Fig. 15 and Fig. 16 describe the phase and amplitude distribution of the 1st 6 IMFs of Nifty and Dow Jones.





(a) (b)

Figure 14. Plots for Amplitude and Phase of the first IMFs for (**a)** Nifty and **(b)** Dow Jones.

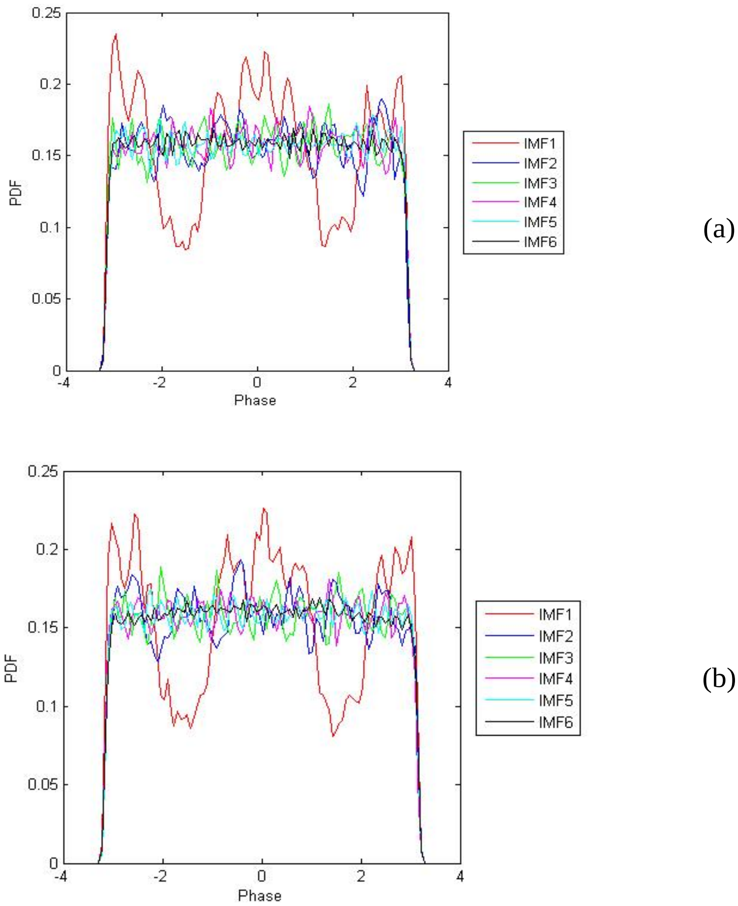
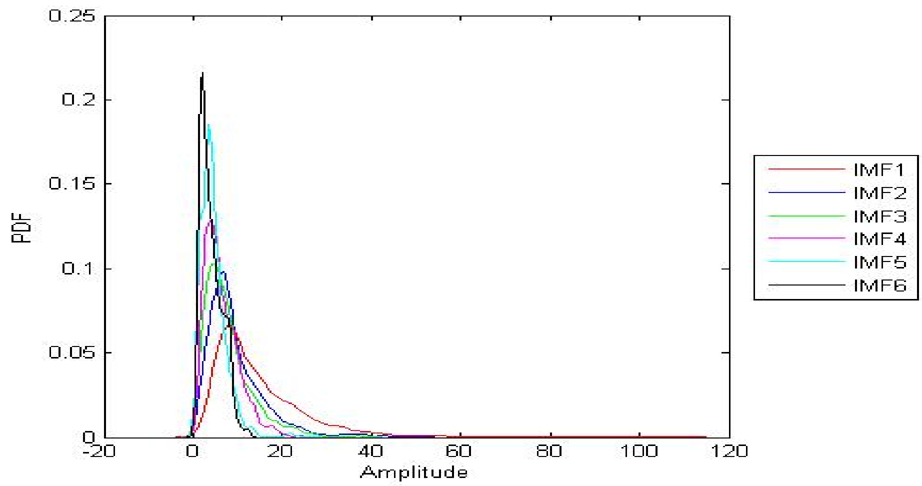
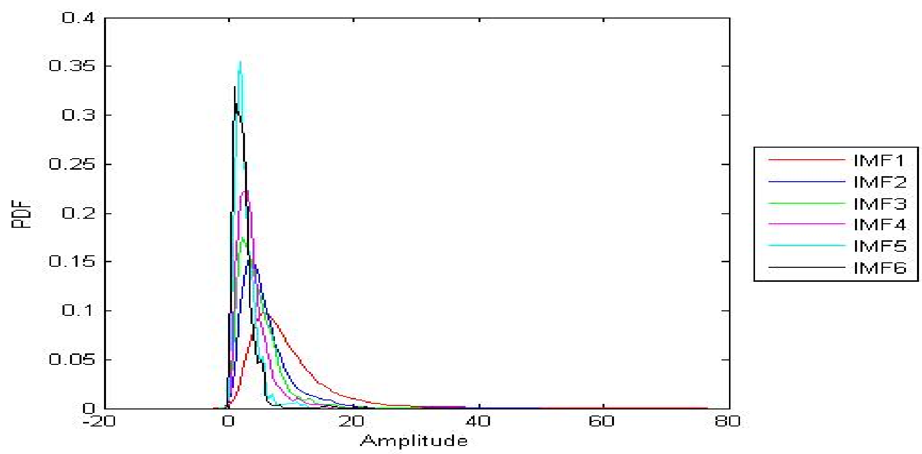


Figure 15. Plots for probability distributions of phases of 1st 6 IMFs of (**a)** Nifty and **(b)** Dow Jones.

We check that the values of the phase lie between – 3.14 to 3.14 in these distributions also.



(a)



(b)

Figure 16. Plots for probability distributions of amplitudes of 1st 6 IMFs of (**a)** Nifty and **(b)** Dow Jones.

Again, from Fig. 7a and b we observe that range of values of amplitude is much higher in Nifty than Dow Jones. Next we have presented a comparative study of the respective phase distributions of Nifty and Dow Jones for the first 6 IMFs with best fitting polynomial regression curves of order 9. Fig. 17 illustrates the results.

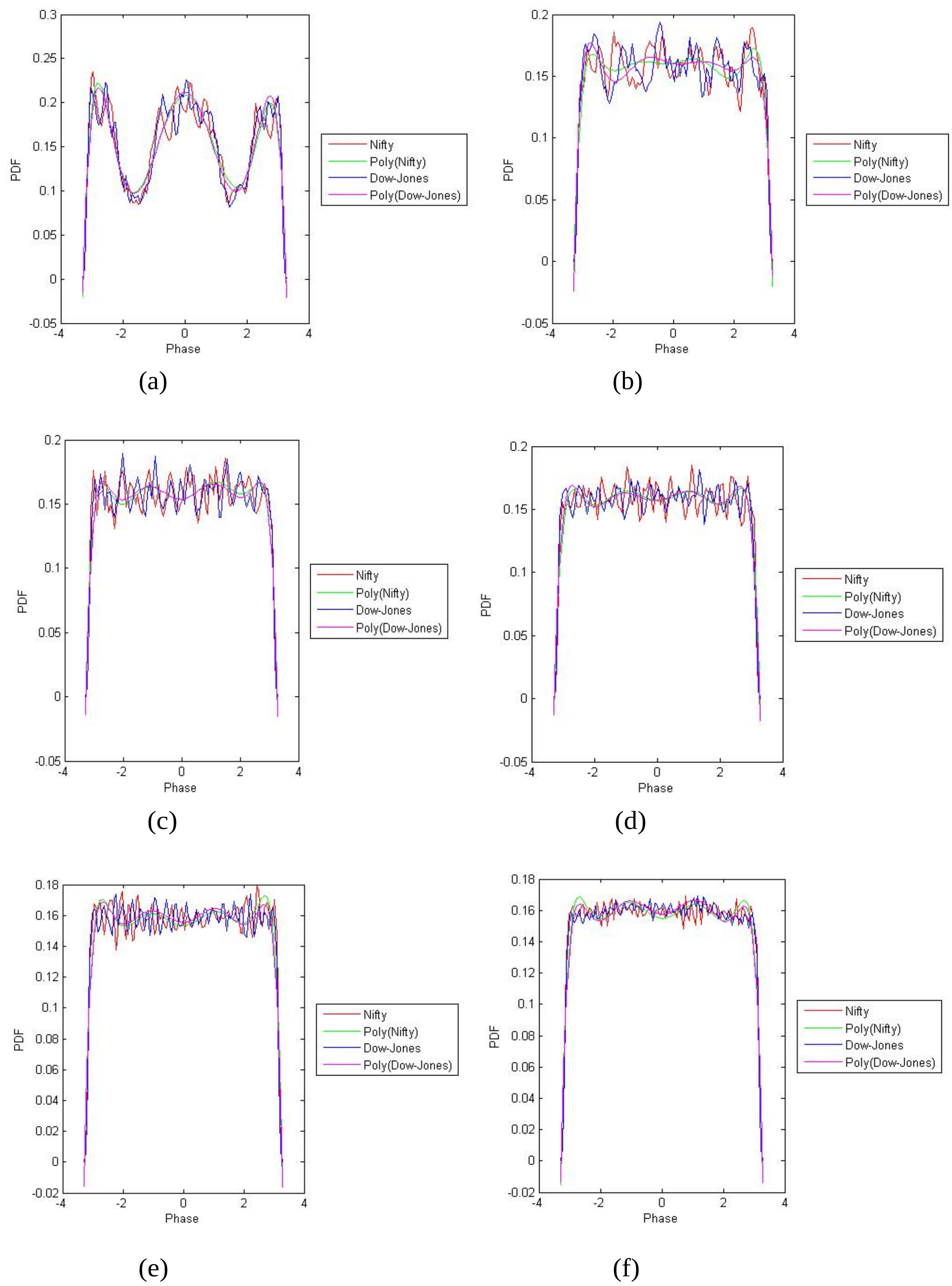


Figure 17. Plots for probability distributions of phases of 1st 6 IMFs of Nifty and Dow Jones with polynomial regression curves of order 9.

Lastly, we have calculated normalized amplitude distribution and it is described in Figure 18.

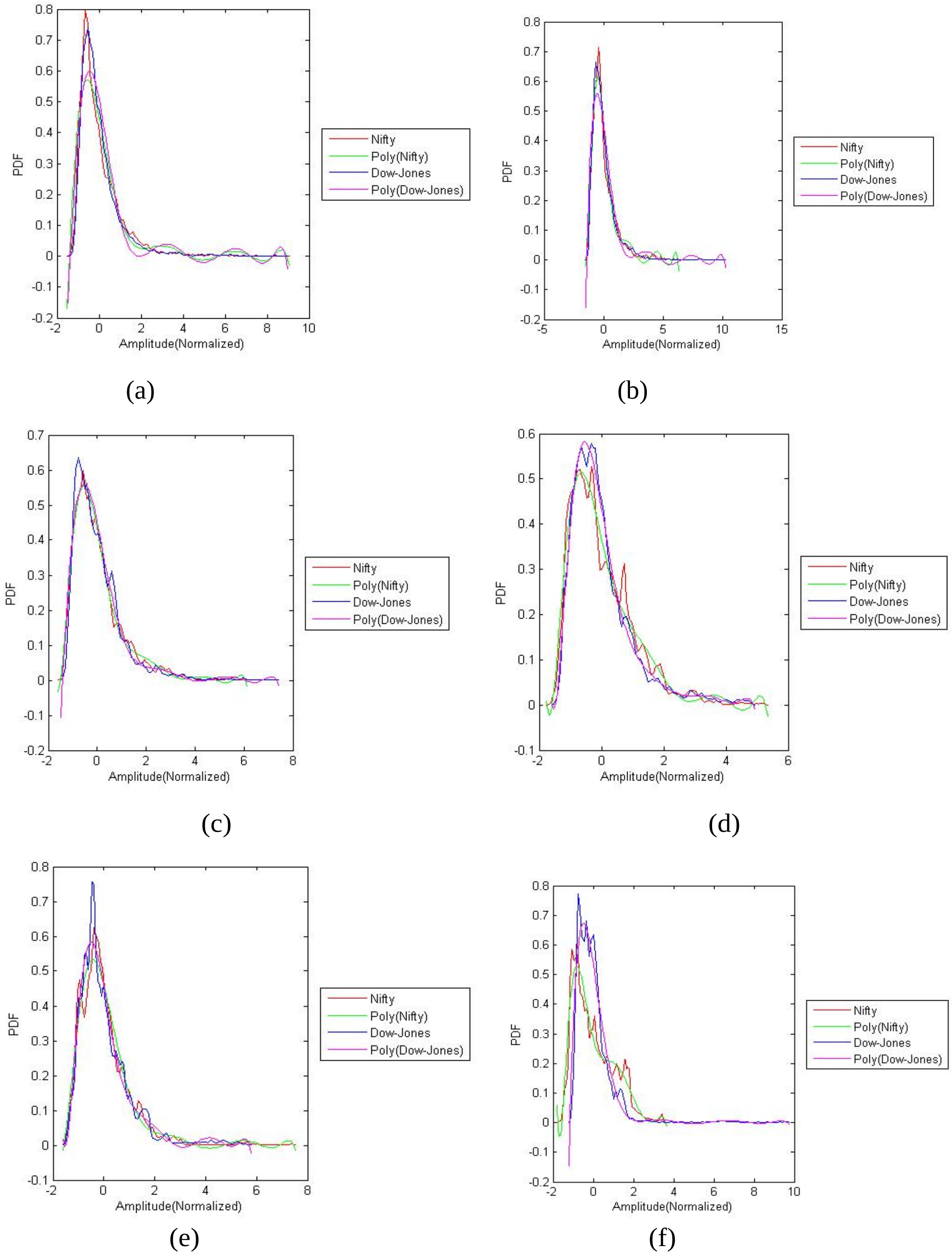


Figure 18. Plots for probability distributions of amplitudes (normalized) of 1st 6 IMFs of Nifty and Dow Jones depicting polynomial regression curves of order 9.

3. Discussion and conclusion

In the present work, we have analyzed two prime Indian stock exchange indices viz. Sensex and Nifty and a well-known American stock exchange index Dow Jones and compared the behaviour of latter one with each of the previous two.

From Fig. 2 and Fig 11, it is evident that the time series are proportional to the sample time scale and though the time series are of two different countries, proportionally is of almost the same order.

From Fig. 3 and Fig. 12, it is seen that there is striking similarity in pattern and behaviour of probability distributions of normalized returns of Dow Jones with Sensex and Nifty. Also, the range of values for the distributions is narrow.

Next when we have performed EMD analysis, we have observed from Fig. 4 and Fig. 13 that all IMFs are independent and orthogonal to each other, i.e. one IMF cannot be generated from other IMFs decomposed from the same primary time series. IMFs mainly differ for intermittencies they inherit. c1 is the 1st mode extracted from $R(t)$, it has the highest frequency among all IMFs. Also it is clear from the Figures that c_1 catches main structure of $R(t)$ which supports the fact that $R(t)$ is mainly characterized by its highest frequency component. Although we should mark that this may not be true for other time series. We should note that c_1 is not equal to $R(t)$. That implies if there are some specified quantities defined for $R(t)$, c_i may produce wrong result.

Then we have analysed instantaneous phase and amplitude. From Fig. 6 and Fig. 15, we have observed that all the values of phases lie between – 3.14 to 3.14 for Sensex, Nifty and Dow Jones. This indicates a striking similarity of the underlying periodicity of Dow Jones with both Sense and Nifty. It is certainly a strong cause to believe that Dow Jones has significant nonlinear correlation with Indian stock markets. The phase distribution is random and they change behaviour abruptly. This is a indication of non-predictable and stochastic features of related indices. Also, from Fig. 7 and Fig. 16, it is shown that range of amplitude values are much higher in both Sensex and Nifty than Dow Jones. This indicates a higher energy level for Indian stock markets than Dow Jones.

From comparative study of phase distribution for respective pairs in Fig. 8 and Fig. 17, it is observed that not only range of values match, but polynomial regression lines are also almost identical. This implies high degree of nonlinear correlation of Dow Jones with Sensex and Nifty. This result emphasises our previously obtained results. Normalized amplitude graphs in Fig. 9 and Fig. 18 also strengthen our initial findings as we finds striking similarity in them.

So, in our work we have found enough evidence to conclude that there are very much similarity and non-linear correlation between Dow Jones and Sensex and Dow Jones and Nifty. We have taken time scale of 1 day in our calculation. Inspired by the findings, we should work for higher time scale in future. Also, we have used interday stock market close data in our work. In future we wish to work with intraday data for micro level analysis of behaviour of stock markets.

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